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A New Approach for Solving Unbalanced Fuzzy Transportation Problems

A. Edward Samuel and P. Raja

Ramanujan Research Centre, PG & Research, Department of Mathematics
Government Arts College (Autonomous), Kumbakonam
Tamil Nadu, India

Abstract

In this paper, new methodologies are proposed for solving unbalanced fuzzy transportation problems without converting into a balanced one. The advantages of these methodologies are to get an optimal solution. In the proposed method, transportation costs, demand and supply are represented by triangular fuzzy numbers. To illustrate the proposed method, numerical examples are solved and the obtained results are compared with the results of other existing approaches. The proposed method is very easy to understand and it can be applied on real life transportation problems for the decision makers.

Mathematics Subject Classification: 90B06, 90C08

Keywords: unbalanced fuzzy transportation problem, Optimization

1. Introduction

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes to stronger. How and when to send the products to the customers in the quantities, they want in a cost effective manner, become more challenging. Transportations models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

The basic transportation problem was originally developed by Hitchcock [6]. We can find initial basic feasible solution by using Vogel's Approximation Method VAM [2]. Many workers describe modifications to Vogel's Approximation method for obtaining initial solutions to the unbalanced transportation problem. Shimshak [12] propose a modification (SVAM) which ignores any penalty that involves a dummy row/column. For example, if there is a dummy column in the cost matrix, the penalties are ignored not only for the dummy column, but also for all the rows since the calculation of row penalties involves the dummy column. Goyal [5] suggests another modification in (GVAM) where the cost of transporting goods to or from a dummy point is set equal to the highest transportation cost in the problem, rather than to zero. The method proposed by Ramakrishnan [11] consists of four steps of reduction and one step of VAM. Balakrishnan [9] suggest further modification in SVAM. K. Jaikumar [7] TOC (Total Opportunity Cost Matrix) using the VAM procedure. They coupled VAM with total opportunity cost and achieved by initial solutions (MNNMPM). Several sorts of methods have been established for finding the optimal solution. Among them some methods have been introduced which directly attain the optimal solution namely zero suffix method [13], ASM – Method [1] etc. But these two methods for finding optimal solution of a transportation problem do not reflect optimal solution proved by Mohammed [8]. In general the transportations problems are solved with the assumptions that the coefficients or cost parameters are specified in precise way ie., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. Zimmerman [14] showed that solutions obtained fuzzy linear programming method is always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Edward Samuel [4] showed improved zero point method (IZPM), is used for solving unbalanced fuzzy transportation problems. Nagoor Gani and Mohamed Assarudeen [10], Proposed a New operation on triangular fuzzy number for solving linear programming problem.

In this study, basic idea is to get an optimal solution for an unbalanced fuzzy transportation problem without converting into a balanced one. This paper presents a new approach, simple and easy to understand for solving unbalanced fuzzy transportation problems. The algorithm of the approach is detailed with suitable numerical examples. Further comparative studies of the new technique with other existing algorithms are established by means of sample problems.

This paper is organized as follows: In section 2, preliminaries and basic definitions are presented. In section 3, a new method is presented to finding an optimal solution. In section 4, a numerical example is solved. The conclusion is discussed in section 5.

2. Preliminaries

In this section, some basic preliminaries are given below

2.1. Definition

A fuzzy number Ã is denoted as a triangular fuzzy number by (a_1, a_2, a_3) and its membership function $\mu_{\tilde{A}}(x)$ is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & if \quad a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & if \quad a_2 \le x \le a_3 \\ 0 & , \quad otherwise \end{cases}$$

2.2. Arithmetic Operations [10]. In this section, arithmetic operations between two triangular fuzzy numbers, defined on the universal set of real numbers $\Re$ are presented.

If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the following is obtained.

(i) $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

(iii) $\tilde{A} \times \tilde{B} = (a, b, c)$ where $T=\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $a=\min\{T\}$, $b=a_2b_2$ and $c=\max\{T\}$

(iv) $\tilde{A} \div \tilde{B} = \left(\dfrac{a_1}{b_1}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_3} \right)$

2.3. Graded mean integration method [3]

The graded mean integration method is used to defuzzify the triangular fuzzy number. The representation of triangular fuzzy number is $\tilde{A} = (a_1, a_2, a_3)$ and its defuzzified value is obtained by $R(A) = \dfrac{(a_1 + 4a_2 + a_3)}{6}$

3. The proposed method

The new methods for solving unbalanced fuzzy transportation problems provide us with an efficient method of finding the optimal solution without converting into a balanced one.

The new methods can be summarized in the following steps:

Step 1. Verify if the total demand (TD) exceeds total supply (TS) i.e, TD > TS
Then go to methodology -I if else go to methodology –II

Methodology –I

Step 2. (a) Locate the smallest element in each column of the given fuzzy cost table and then subtract that from each element of that column, and

(b) In the reduced matrix obtained from 1(a), locate the smallest element in each row and then subtract that from each element of that row.

Step 3. Calculate penalties for each row by taking the difference between the smallest and next smallest unit fuzzy transportation cost in the same row.

Step 4. Select the row with the largest penalty and allocate as much as possible in the cell having the least cost in the selected row satisfying the rim conditions.

Step 5. Repeat step 3 to 4 until the entire demand at various destinations or available supply at various sources is satisfied.

Step 6. Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

Methodology –II

Step 1. (a) Locate the smallest element in each row of the given fuzzy cost table and then subtract that from each element of that row, and

(b) In the reduced matrix obtained from 1(a), locate the smallest element in each column and then subtract that from each element of that column.

(c) Select the smallest fuzzy odd cost in the reduced matrix obtained from 1(b) and then subtract that from selected fuzzy odd costs only. If there is no fuzzy odd costs then select the smallest fuzzy even cost and divide the selected even fuzzy cost only.

Step 2. Calculate penalties for each column by taking the difference between the smallest and next smallest unit fuzzy transportation cost in the same column.

Step 3. Select the column with the largest penalty and allocate as much as possible in the cell having the least cost in the selected column satisfying the rim conditions

Step 4. Repeat step 8 to 9 until the entire demand at various destinations or available supply at various sources is satisfied.

Step 5. Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

Important Remarks

1. If there is a tie in the minimum fuzzy transportation cost, then select the smallest cell for allocation from the original fuzzy cost table.
2. If there is a tie in the values of penalties then it can be broken by the smallest fuzzy odd cost cell from the original fuzzy cost table.

4. Numerical Examples

To illustrate the proposed method, the following unbalanced fuzzy transportation problem is solved without converting into balanced one.

4.1. Problem (Methodology I)

A company has three manufacturing plants and three warehouses. Each plants manufactures the same product which is solid at different prices in each warehouses area. The capacities of the plants are also different. The uncertain transportation cost, demand and supply are given in the following table.

	D_1	D_2	D_3	Supply(a_i)
S_1	(4,5,6)	(0,1,2)	(5,7,9)	(9,10,11)
S_2	(4,6,8)	(2,4,6)	(4,6,8)	(79,80,81)
S_3	(2,3,4)	(5,7,9)	(4,5,6)	(14,15,16)
Demand(b_j)	(74,75,76)	(19,20,21)	(49,50,51)	

Since $\sum_{i=1}^{3} \tilde{a}_i = (102,105,108) \neq \sum_{j=1}^{3} \tilde{b}_j = (142,145,148)$ so the chosen problem is a unbalanced fuzzy transportation problem.

Using step 1(i.e, TD > TS) , 2(a) & (b), we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(2,2,2)	(0,0,0)	(1,2,3)	(9,10,11)
S_2	(2,2,2)	(2,2,2)	(0,0,0)	(79,80,81)
S_3	(0,0,0)	(5,6,7)	(0,0,0)	(14,15,16)
Demand(b_j)	(74,75,76)	(19,20,21)	(49,50,51)	

Using step 3, we get

	D_1	D_2	D_3	Supply(a_i)	Row penalty
S_1	(2,2,2)	(0,0,0)	(1,2,3)	(9,10,11)	(1,2,3)
S_2	(2,2,2)	(2,2,2)	(0,0,0)	(79,80,81)	(2,2,2)
S_3	(0,0,0)	(5,6,7)	(0,0,0)	(14,15,16)	(0,0,0)
Demand(b_j)	(74,75,76)	(19,20,21)	(49,50,51)		

Using step 4 to step 5, we get

	D_1	D_2	D_3	Supply(a_i)	Row penalty
S_1	*	(0,0,0) **(9,10,11)**	*	(9,10,11)	**(1,2,3)**
S_2	(2,2,2) **(20,20,20)**	(2,2,2) **(10,10,10)**	(0,0,0) **(49,50,51)**	(79,80,81)	(2,2,2)
S_3	(0,0,0) **(14,15,16)**	*	*	(14,15,16)	(0,0,0)
Demand(b_j)	(74,75,76)	(19,20,21)	(49,50,51)		

Using step 6, we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(4,5,6)	(0,1,2) **(9,10,11)**	(5,7,9)	(9,10,11)
S_2	(4,6,8) **(20,20,20)**	(2,4,6) **(10,10,10)**	(4,6,8) **(49,50,51)**	(79,80,81)
S_3	(2,3,4) **(14,15,16)**	(5,7,9)	(4,5,6)	(14,15,16)
Demand(b_j)	(74,75,76)	(19,20,21)	(49,50,51)	

The minimum fuzzy transportation cost is

= (0,1,2) (9,10,11) + (4,6,8) (20,20,20) + (2,4,6)(10,10,10) + (4,6,8) (49,50,51)
+ (2,3,4)(14,15,16)
= (324,515,714)

R(A) = 516.33

4.2. Problem (Methodology II)

A cement product is manufactured at four manufacturing centers S_1, S_2, S_3 and S_4 to depots D_1, D_2 and D_3. A production manager is considering the best way to transport cement centers to depots. The uncertain weekly production and demands along with transportation cost are given below:

	D_1	D_2	D_3	Supply(a_i)
S_1	(1,2,3)	(5,7,9)	(12,14,16)	(4,5,6)
S_2	(1,3,5)	(1,3,5)	(0,1,2)	(6,8,10)
S_3	(3,5,7)	(2,4,6)	(5,7,9)	(6,7,8)
S_4	(0,1,2)	(4,6,8)	(1,2,3)	(13,15,17)
Demand(b_j)	(5,7,9)	(8,9,10)	(16,18,20)	

Since $\sum_{i=1}^{4} \tilde{a}_i = (29,35,41) \neq \sum_{j=1}^{3} \tilde{b}_j = (29,34,39)$, so the chosen problem is a unbalanced fuzzy transportation problem.

Using step 1(i.e, TD < TS), 1(a) & 1(b), we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(0,0,0)	(4,5,6)	(11,12,13)	(4,5,6)
S_2	(1,2,3)	(1,2,3)	(0,0,0)	(6,8,10)
S_3	(1,1,1)	(0,0,0)	(3,3,3)	(6,7,8)
S_4	(0,0,0)	(4,5,6)	(1,1,1)	(13,15,17)
Demand(b_j)	(5,7,9)	(8,9,10)	(16,18,20)	

Using step 1(c), we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(0,0,0)	(3,4,5)	(11,12,13)	(4,5,6)
S_2	(1,2,3)	(1,2,3)	(0,0,0)	(6,8,10)
S_3	(0,0,0)	(0,0,0)	(2,2,2)	(6,7,8)
S_4	(0,0,0)	(3,4,5)	(0,0,0)	(13,15,17)
Demand(b_j)	(5,7,9)	(8,9,10)	(16,18,20)	

Using step 2, we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(0,0,0)	(3,4,5)	(11,12,13)	(4,5,6)
S_2	(1,2,3)	(1,2,3)	(0,0,0)	(6,8,10)
S_3	(0,0,0)	(0,0,0)	(2,2,2)	(6,7,8)
S_4	(0,0,0)	(3,4,5)	(0,0,0)	(13,15,17)
Demand(b_j)	(5,7,9)	(8,9,10)	(16,18,20)	
Column penalty	**(0,0,0)**	**(1,2,3)**	**(0,0,0)**	

Using step 3, we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(0,0,0)	(3,4,5)	(11,12,13)	(4,5,6)
S_2	(1,2,3)	(1,2,3)	(0,0,0)	(6,8,10)
S_3	*	(0,0,0) **(6,7,8)**	*	*
S_4	(0,0,0)	(3,4,5)	(0,0,0)	(13,15,17)
Demand(b_j)	(5,7,9)	(2,2,2)	(16,18,20)	
Column penalty	(0,0,0)	**(1,2,3)**	(0,0,0)	

Using step 4, we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(0,0,0) **(4,4,4)**	(3,4,5)	(11,12,13)	(4,5,6)
S_2	(1,2,3)	(1,2,3) **(2,2,2)**	(0,0,0) **(4,6,8)**	(6,8,10)
S_3	(0,0,0)	(0,0,0) **(6,7,8)**	(2,2,2)	(6,7,8)
S_4	(0,0,0) **(1,3,5)**	(3,4,5)	(0,0,0) **(12,12,12)**	(13,15,17)
Demand(b_j)	(5,7,9)	(8,9,10)	(16,18,20)	

Using step 5, we get

	D_1	D_2	D_3	Supply(a_i)
S_1	(1,2,3) **(4,4,4)**	(5,7,9)	(12,14,16)	(4,5,6)
S_2	(1,3,5)	(1,3,5) **(2,2,2)**	(0,1,2) **(4,6,8)**	(6,8,10)
S_3	(3,5,7)	(2,4,6) **(6,7,8)**	(5,7,9)	(6,7,8)
S_4	(0,1,2) **(1,3,5)**	(4,6,8)	(1,2,3) **(12,12,12)**	(13,15,17)
Demand(b_j)	(5,7,9)	(8,9,10)	(16,18,20)	

The minimum fuzzy transportation cost is

= (1,2,3) (4,4,4) + (1,3,5) (2,2,2) + (0,1,2)(4,6,8) + (2,4,6) (6,7,8) + (0,1,2)(1,3,5)
+ (1,2,3) (12,12,12)
= (30, 75, 132)

R(A) = 77.00

Sl.No.	Row	Col	VAM	SVAM	GVAM	BVAM	RVAM	ASM	ZSM	MNNMPM	IZPM	MODI	Proposed method
1.	**3**	**3**	**516.33**	**516.33**	**516.33**	**516.33**	**516.33**	**516.33**	**516.33**	536.33	**516.33**	**516.33**	**516.33**
2.	**4**	**3**	84.00	103.33	81.33	81.33	95.00	95.00	81.00	81.33	**77.00**	**77.00**	**77.00**

5. Conclusion

The proposed method has the following major advantages,

(i) Unbalanced fuzzy transportation problems are solved, without converting into a balanced one

(ii) Very easy to understand

(iii) Better than the existing methods and

(iv) The solution is always optimal.

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