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Slightly Fuzzy ω-Continuous Mappings

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Abstract

In this paper slightly fuzzy ω-continuous mappings is introduced and discussions on some interesting properties and characterizations of slightly fuzzy continuous mappings are done.

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1. Introduction

The fuzzy concept has penetrated almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [10]. Fuzzy sets have applications in many fields such as information [7] and control [9]. The theory of fuzzy topological spaces was introduced and developed by Chang [3]. The concept of slightly fuzzy continuous mappings was introduced by Sudha, Roja and Uma [8]. The concept of ω-continuity in topological spaces was introduced by Sheik John [6]. The motivation of this paper is to introduce slightly fuzzy ω-continuous mappings. Some interesting properties and characterizations of these mappings are discussed with necessary examples. In this paper, almost* fuzzy ω-continuous mappings, θ*-fuzzy ω-continuous mappings and weakly* fuzzy ω-continuous mappings are introduced and studied. It is also observed that slightly fuzzy ω-continuous mappings preserve fuzzy ω-connectedness and every slightly fuzzy ω-continuous mapping into a fuzzy ω-extremally disconnected space is almost* fuzzy ω-continuous.

2. Preliminaries

We recall the following definitions which we used in this paper.

Definition 2.1 [8]

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) $\rightarrow$ (Y, S) is said to be almost* fuzzy continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \text{int cl } \mu$.

Definition 2.2 [8]

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) $\rightarrow$ (Y, S) is said to be θ*-fuzzy continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\text{cl } \sigma) \leq \text{cl } \mu$.

Definition 2.3 [8]

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T)$\rightarrow$ (Y, S) is said to be weakly* fuzzy continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \text{cl } \mu$.

Definition 2.4 [8]

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) $\rightarrow$ (Y, S) is said to be slightly fuzzy continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy clopen set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mu$.

Definition 2.5 [5]

Let (D, $\geq$) be a directed set. Let X be an ordinary set. Let f be the collection of all fuzzy points in X. The function S : D $\rightarrow f$ is called a fuzzy net in X. In otherwords, a fuzzy net is a pair (S, $\geq$) such that S is a function : D $\rightarrow f$ and $\geq$ directs the domain of S. For $n \in D$, S (n) is often denoted by S_n and hence a net S is often denoted by $\{ S_n, n \in D \}$.

Definition 2.6 [3]

A sequence of fuzzy sets, say $\{ A_n; n = 1, 2, ... \}$, is eventually contained in a fuzzy set A iff there is an integer m such that, if $n \geq m$, then $A_n \subset A$.

Definition 2.7 [1]

Let (X, T) and (Y, S) be any two fuzzy topological spaces. For a mapping f : (X, T) $\rightarrow$ (Y, S), the graph g : X $\rightarrow$ X $\times$ Y of f is defined by g (x) = (x, f (x)), for each $x \in X$.

Definition 2.8 [4]

A fuzzy topological (X, T) is said to be fuzzy connected iff the only fuzzy sets which are both fuzzy open and fuzzy closed are 0 and 1.

Definition 2.9 [2]

A fuzzy topological space (X, T) is said to be fuzzy extremally disconnected if the fuzzy closure of every fuzzy open set is fuzzy open.

3. Some properties and characterizations of slightly fuzzy ω-continuous mappings

In this section we investigate some properties of slightly fuzzy ω-continuous mappings and we also obtain characterizations of these mappings.

Definition 3.1

Let (X, T) be a topological space. A fuzzy set $\lambda \in I^X$ is called a fuzzy ω-closed set in (X, T) if cl (λ) ≤ μ whenever λ ≤ μ and μ is fuzzy semi-open in (X, T).

The complement of a fuzzy ω-closed set in (X, T) is fuzzy ω-open.

Notation 3.1

(a) ω-cl (λ) denotes fuzzy ω-closure of λ.

(b) ω-int (λ) denotes fuzzy ω-interior of λ.

Definition 3.2

A mapping f : (X, T) → (Y, S) is called

(a) fuzzy ω-continuous if f^{-1} (λ) is fuzzy ω-closed in (X, T) for every fuzzy closed set λ in (Y, S).

(b) fuzzy ω-irresolute if f^{-1} (λ) is fuzzy ω-closed in (X, T) for every fuzzy ω-closed set λ in (Y, S).

Definition 3.3

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) → (Y, S) is said to be almost* fuzzy ω-continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with f (α) ≤ μ, there exists a fuzzy ω-open set σ with α ≤ σ such that f (σ) ≤ int (cl (μ)).

Definition 3.4

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) → (Y, S) is said to be θ*-fuzzy ω-continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with f (α) ≤ μ, there exists a fuzzy ω-open set σ with α ≤ σ such that f (cl σ) ≤ cl (μ).

Definition 3.5

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) → (Y, S) is said to be weakly* fuzzy ω-continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with f (α) ≤ μ, there exists a fuzzy ω-open set σ with α ≤ σ such that f (σ) ≤ (cl (μ).

Definition 3.6

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) → (Y, S) is said to be slightly fuzzy ω-continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy clopen set μ with f (α) ≤ μ, there exists a fuzzy ω-open set σ with α ≤ σ such that f (σ) ≤ μ.

Remark 3.1

Every weakly fuzzy ω-continuous mapping is slightly fuzzy ω-continuous obviously.

Hence fuzzy ω-continuity $\Rightarrow$ almost* fuzzy ω-continuity $\Rightarrow$ θ*-fuzzy ω-continuity $\Rightarrow$ weak* fuzzy ω-continuity $\Rightarrow$ slight fuzzy ω-continuity.

But none is reversible as shown in the following examples.

Example 3.1

Let X = { a, b, c }. Define T_1 = { 0, 1, λ_1, λ_2 }, T_2 = { 0, 1, μ, γ, δ_1, δ_2 } where λ_1, λ_2, μ, γ, δ_1, δ_2 : X $\rightarrow$ [0, 1] are such that λ_1 (a) = 0.3, λ_1 (b) = 0.4, λ_1 (c) = 0.5, λ_2 (a) = 0.7, λ_2 (b) = 0.6, λ_2 (c) = 0.5, μ (a) = 0.5, μ (b) = 0.5, μ (c) = 0.5, γ (a) = 1, γ (b) = 0.5, γ (c) = 0.5, δ_1 (a) = 0, δ_1 (b) = 0, δ_1 (c) = 0.3 and δ_2 (a) = 1, δ_2 (b) = 0, δ_2 (c) = 0.3. Clearly (X, T_1) and (X, T_2) are fuzzy topological spaces. Define f : (X, T_1) $\rightarrow$ (X, T_2) as f (a) = b, f (b) = a and f (c) = c. Let λ, α : X $\rightarrow$ [0, 1] be any fuzzy sets such that α (a) = 0, α (b) = 0, α (c) = 0.1 and λ (a) = 0.4, λ (b) = 0.4, λ (c) = 0.5. For the fuzzy clopen set μ in (X, T_2), f (α) $\leq$ μ. Now λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (λ) $\leq$ μ. **Hence f is slightly fuzzy ω-continuous**.

f (α) $\leq$ δ_1 where δ_1 is a fuzzy open set in (X, T_2). But λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (λ) $\not\leq$ cl (δ_1). **Hence f is not weakly* fuzzy ω-continuous**.

Example 3.2

Let X = { a, b, c }. Define T_1 = { 0, 1, δ_1, δ_2 } and T_2 = { 0, 1, μ, γ } where δ_1, δ_2 μ, γ : X $\rightarrow$ [0, 1] are such that δ_1 (a) = 0.3, δ_1 (b) = 0, δ_1 (c) = 0.5, δ_2 (a) 0.7, δ_2 (b) = 1, δ_2 (c) = 0.5, μ (a) = 0.3, μ (b) = 0.3, μ (c) = 0.3 and γ (a) = 1, γ (b) = 0.5, γ (c) = 0.5. Clearly (X, T_1) and (X, T_2) are fuzzy topological spaces. Define f : (X, T_1) $\rightarrow$ (X, T_2) as f (a) = b, f (b) = a and f (c) = c. Let λ, α : X $\rightarrow$ [0,1] be any fuzzy sets such that α (a) = 0, α (b) = 0, α (c) = 0. 1 and λ (a) = 0.5, λ (b) = 0.5, λ (c) = 0.5. Then for every fuzzy open set ρ in (X, T_2) with f (α) $\leq \rho$, λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (λ) $\leq$ cl (ρ). **Hence f is weakly* fuzzy ω-continuous**.

f (α) $\leq$ μ where μ is a fuzzy open set in (X, T_2). But λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (cl (λ)) $\not\leq$ cl (μ). **Hence f is not θ*-fuzzy ω-continuous**.

Example 3.3

Let X = { a, b, c }. Define T_1 = { 0, 1, λ_1, λ_2 } and T_2 = { 0, 1, ρ } where λ_1, λ_2, ρ : X $\rightarrow$ [0, 1] are such that λ_1 (a) = 0.3, λ_1 (b) = 0.4, λ_1 (c) = 0.5, λ_2 (a) = 0.7, λ_2 (b) = 0.6, λ_2 (c) = 0.5 and ρ (a) = 0, ρ (b) = 0, ρ (c) = 0.5. Clearly (X, T_1) and (X, T_2) are fuzzy topological spaces. Define f : (X, T_1) $\rightarrow$ (X, T_2) as f (a) = b, f (b) = a, f (c) = c. Let α, λ : X $\rightarrow$ [0, 1] be any fuzzy sets such that α (a) = 0, α (b) = 0, α (c) = 0.2 and λ (a) = 0.5, λ (b) = 0.5, λ (c) = 0.5. Then for every fuzzy open set ρ in (X, T_2) with f (α) $\leq \rho$, λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (cl (λ)) $\leq$ cl (ρ). **Therefore f is θ*-fuzzy ω-continuous**.

λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (λ) $\not\leq$ int (cl (ρ)). **Therefore f is not almost* fuzzy ω-continuous**.

Example 3.4

Let X = { a, b }. Define T_1 = { 0, 1, λ_1, λ_2, λ_3 } and T_2 = { 0, 1, μ_1, μ_2 } where λ_1, λ_2, λ_3, μ_1, μ_2 : X → [0, 1] are such that λ_1 (a)= 0.9, λ_1 (b) = 0.7, λ_2 (a) = 1, λ_2 (b) = 0.9, λ_3 (a) = 0.11, λ_3 (b) = 0.31, μ_1 (a) = 0, μ_1 (b) = 0.2 and μ_2 (a) = 0.75, μ_2 (b) = 0.75. Clearly (X, T_1) and (X, T_2) are fuzzy topological spaces. Let f : (X, T_1) → (X, T_2) be the identify function. Let α, λ : X → [0,1] be any fuzzy sets such that α (a) = 0, α (b) = 0.1 and λ (a) = 0.7, λ (b) = 0.4. For the fuzzy open set μ_1 in (X, T_2) with f (α) ≤ μ_1, λ is a fuzzy ω-open set in (X, T_1) with $\alpha \leq \lambda$ such that f (λ) ≤ int (cl (μ_1)). **Hence f is almost* fuzzy ω-continuous**.

For the fuzzy open μ_1 in (X, T_2), f^{-1} (μ_1) is not fuzzy ω-open. **Therefore f is not fuzzy ω-continuous**.

Proposition 3.1

Let (X, T_1) and (Y, T_2) be any two fuzzy topological spaces. For a mapping f : (X, T_1) → (X, T_2) the following conditions are equivalent:

(a) f is slightly fuzzy ω-continuous.

(b) Inverse image of every fuzzy ω-clopen set of (Y, T_2) is a fuzzy ω-open set of (X, T_1).

(c) Inverse image of every fuzzy ω-clopen set of (Y, T_2) is a fuzzy ω-clopen set of (X, T_1).

(d) For each fuzzy set $\alpha \in I^X$ and for every fuzzy net { S_n,n ∈ D } which converges to α, the fuzzy net { f (S_n), n ∈ D } is eventually in each fuzzy ω-clopen set λ with f (α) ≤ λ.

Proof (a) ⇒ (b) Let ρ be a fuzzy ω-clopen set of (Y, T_2) and let $\lambda \in I^X$ be any fuzzy set such that $\lambda \leq f^{-1}(\rho)$. Now, ρ is a fuzzy ω-clopen set with f (λ) ≤ ρ. Hence by (a), there exists a fuzzy ω-open set μ of (X, T_1) with $\lambda \leq \mu$ such that f (μ) ≤ ρ. Hence f^{-1} (ρ) is a fuzzy ω-open set of (X, T_1).

(b) ⇒ (c) Let μ be a fuzzy ω-clopen set of (Y, T_2). Now, 1 – μ is a fuzzy ω-clopen set of (Y, T_2). Therefore by (b), f^{-1} (1 - μ) = 1– f^{-1} (μ) is a fuzzy ω-open set of (X, T_1). That is, f^{-1}(μ) is a fuzzy ω-closed set of (X, T_1). By (b), f^{-1} (μ) is a fuzzy ω-open set of (X, T_1). Thus f^{-1} (μ) is a fuzzy ω-clopen set of (X, T_1).

(c) ⇒ (d) Let { S_n, n ∈ D } be a fuzzy net converging to a fuzzy set α and let β be a fuzzy ω-clopen set with f (α) ≤ β. By (c), there exists a fuzzy ω-open set λ with $\alpha \leq \lambda$ such that f (λ) ≤ β. Since the net { S_n, n ∈ D } converges to α implies $S_n \leq \alpha$, now, $S_n \leq \alpha \leq \lambda$. Thus f (S_n) ≤ f (λ) ≤ β. Hence { f (S_n), n ∈ D } is eventually in β.

(d) ⇒ (a) Suppose that f is not slightly fuzzy ω-continuous. Then there does not exist a fuzzy ω-open set λ with $\alpha \leq \lambda$, such that f (λ) ≤ μ and hence

$f(S_n) \leq \mu$. This implies that the fuzzy net $\{ f(S_n), n \in D \}$ is not eventually in a fuzzy ω-clopen set μ with $f(\alpha) \leq \mu$, which is a contradiction. Hence f is slightly fuzzy ω-continuous.

Proposition 3.2

Let (X, T) be any fuzzy topological space and let A be a subspace of X. Then the inclusion map $j : (A, T / A) \to (X, T)$ is slightly fuzzy ω-continuous.

Proposition 3.3

Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be slightly fuzzy ω-continuous mappings. Then their composition gof is slightly fuzzy ω-continuous.

Proposition 3.4

Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be a surjective fuzzy ω-open and fuzzy ω-irresolute and let $g : (Y, S) \to (Z, R)$ be any mapping. Then $gof : (X, T) \to (Z, R)$ is slightly fuzzy ω-continuous iff g is slightly fuzzy ω-continuous.

Proof

Suppose that gof is slightly fuzzy ω-continuous. Let λ be a fuzzy ω-clopen set of (Z, R). Then by Proposition 3.1, $(gof)^{-1}(\lambda)$ is a fuzzy ω-open in (X, T). That is, $f^{-1}(g^{-1}(\lambda))$ is fuzzy ω-open. Since f is fuzzy ω-open, $f(f^{-1}(g^{-1}(\lambda)))$ is fuzzy ω-open in (Y, S). That is, $g^{-1}(\lambda)$ is fuzzy ω-open in (Y, S). Therefore by Proposition 3.1, g is slightly fuzzy ω-continuous. Conversely let μ be a fuzzy ω-clopen set of (Z, R). By Proposition 3.1, since g is slightly fuzzy ω-continuous, $g^{-1}(\mu)$ is fuzzy ω-open in (Y, S). Since f is fuzzy ω-irresolute, $f^{-1}(g^{-1}(\mu)) = (gof)^{-1}(\mu)$ is fuzzy ω-open in (X, T). Therefore by Proposition 3.1, gof is slightly fuzzy ω-continuous.

Proposition 3.5

Every restriction of a slightly fuzzy ω-continuous mapping is slightly fuzzy ω-continuous.

Proposition 3.6

Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be a mapping. Then the graph of f, $g : X \to X \times Y$ is slightly fuzzy ω-continuous iff f is slightly fuzzy ω-continuous.

Proof

Suppose that $f : (X, T) \to (Y, S)$ is slightly fuzzy ω-continuous and let $g : X \to X \times Y$ be the graph of f. Let $\lambda \times \mu$ be a fuzzy ω-clopen set of $X \times Y$.

$$\begin{aligned} \text{Then } g^{-1}(\lambda \times \mu)(x) &= (\lambda \times \mu)\, g(x) \\ &= (\lambda \times \mu)(x, f(x)) \\ &= \min(\lambda(x), \mu(f(x))) \\ &= (\lambda \wedge f^{-1}(\mu))(x) \end{aligned}$$

$$\text{Therefore } g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu).$$

Since $g^{-1}(\lambda \times \mu)$ is a fuzzy ω-open set of (X, T), by Proposition 3.1, g is slightly fuzzy ω-continuous.

Conversely let λ be fuzzy ω-clopen in (Y, S). Then $1 \times \lambda$ is fuzzy ω-clopen in $X \times Y$. Since g is slightly fuzzy ω-continuous, by Proposition 3.1, $g^{-1}(1\times\lambda)$ is fuzzy ω-open in (X, T). Also $g^{-1}(1 \times \lambda) = f^{-1}(\lambda)$. Therefore $f^{-1}(\lambda)$ is fuzzy ω-open in (X, T). Hence f is slightly fuzzy ω-continuous.

Proposition 3.7

Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be slightly fuzzy ω-continuous. Then the mapping $g : (X, T) \to (Z, R)$ where $R = S / Z$ is slightly fuzzy ω-continuous.

Proposition 3.8

Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces and let $Y \subset Z$ be a subspace of Z. Then the mapping $h : (X, T) \to (Z, R)$ obtained by expanding the range of the slightly fuzzy ω-continuous mapping $f : (X, T) \to (Y, S)$ is slightly fuzzy ω-continuous.

Proposition 3.9

Let $h: X \to \prod_{\alpha\in I} X_\alpha$ be a slightly fuzzy ω-continuous mapping. For each $\alpha\in I$, define $f_\alpha : X \to X_\alpha$ by setting $f_\alpha(\lambda) = (h(\lambda))_\alpha$. Then f_α is slightly fuzzy ω-continuous, for every $\alpha \in I$.

Proof

Let $\delta \in I^X$ and let μ be any fuzzy ω-clopen set in X_α. Let $h(\delta) \le \mu$. Then $f_\alpha(\delta) = (h(\delta))_\alpha \le \mu$. Since h is slightly fuzzy ω-continuous, there exists an ω-open set λ with $\delta \le \lambda$ such that

$$\begin{aligned} & h(\lambda) \le \mu \\ \Rightarrow\ & (h(\lambda))_\alpha \le \mu \\ \Rightarrow\ & f_\alpha(\lambda) \le \mu. \end{aligned}$$

Hence f_α is slightly fuzzy ω-continuous.

Proposition 3.10

Let (X, T), (X, T_1) and (X, T_2) be any three fuzzy topological spaces and let $p_i : X_1 \times X_2 \to X_i$ (i = 1, 2) be the projections of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is a slightly fuzzy ω-continuous mapping then $p_i of$ is also slightly fuzzy ω-continuous.

Proposition 3.11

Let (X, T) and (Y, S) be any two fuzzy topological spaces such that elements of S are both fuzzy ω-open and fuzzy ω-closed. If $f : (X, T) \to (Y, S)$ is slightly fuzzy ω-continuous then f is fuzzy ω-continuous.

Proof

Proof follows from (a) $\Rightarrow$ (b) of Proposition 3.1.

Definition 3.8

A fuzzy topological space (X, T) is said to be fuzzy ω-connected iff the only fuzzy sets which are both fuzzy ω-open and fuzzy ω-closed are 0 and 1.

Proposition 3.12

Every mapping from a fuzzy topological space to a fuzzy ω-connected space is slightly fuzzy ω-continuous.

Proposition 3.13

A slightly fuzzy ω-continuous image of a fuzzy ω-connected space is fuzzy ω-connected.

Proof

Let (X, T) be a fuzzy ω-connected space and (Y, S) be any fuzzy topological space. Let f : (X, T) $\rightarrow$ (Y, S) be a slightly fuzzy ω-continuous mapping. Suppose that (Y, S) is fuzzy ω-disconnected. Let λ be a proper fuzzy ω-clopen set of (Y, S). Since f is slightly fuzzy ω-continuous, by Proposition 3.1, $f^{-1}(\lambda)$ is a proper fuzzy ω-clopen set of (X, T), which is a contradiction. Hence Y is fuzzy ω-connected.

Definition 3.9

Let (X, T) be a fuzzy topological space. (X, T) is called fuzzy ω-extremally disconnected if the fuzzy ω-closure of every fuzzy ω-open set is fuzzy ω-open.

Proposition 3.14

Let (X, T) be a fuzzy topological space and (Y, S) be a fuzzy ω-extremally disconnected space. If f : (X, T) $\rightarrow$ (Y, S) is a slightly fuzzy ω-continuous mapping then f is almost* fuzzy ω-continuous.

Proof

Let μ be a fuzzy ω-open set of (Y, S) and $\lambda \in I^X$ with $f(\lambda) \leq \mu$. Since (Y, S) is fuzzy ω-extremally disconnected, ω-cl (μ) is fuzzy ω-open and therefore fuzzy ω-clopen. Now, $f(\lambda) \leq \omega\text{-cl}(\mu) \leq \text{cl}(\mu)$ and since f is slightly fuzzy ω-continuous, there exists a fuzzy ω-open set σ with $\lambda \leq \sigma$ such that $f(\sigma) \leq \text{cl}(\mu)$. Therefore f is almost* fuzzy ω-continuous.

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