Ideal on Supra Topological Space

Shyamapada Modak and Sukalyan Mistry

Department of Mathematics
University of Gourbanga
Mokdumpur, Malda-732103
West Bengal, India
spmmodak2000@yahoo.co.in

Abstract. In this paper we introduce the ideal on supra topological space and we shall discuss the properties of this space. In this space we introduce two operators \( \mu^\nu \) and \( \psi_\mu \). A generalized set has also been introduced in this space with the help of \( \psi_\mu \) operator.

Mathematics Subject Classification: 54A05, 54C10

Keywords: supra topological space, \( \mu^\nu \) - operator, \( \psi_\mu \) - operator, supra compatible, \( \psi_\mu \) - C set

1. Introduction

The concept of ideal in topological space was first introduced by Kuratowski[5] and Vaidyanathswamy[14]. They also have defined local function in ideal topological space. Further Hamlett and Jankovic in [3] and [4] studied the properties of ideal topological spaces and they have introduced another operator called \( \psi \) operator. They have also obtained a new topology from original ideal topological space. Using the local function, they defined a Kuratowski Closure operator in new topological space. Further, they showed that interior operator of the new topological space can be obtained by \( \psi \) - operator. Modak and Bandyopadhyay[9] in 2007 have defined generalized open sets using \( \psi \) operator. More recently Al-Omri and Noiri[1] have defined the ideal m-space and introduced two operators as like similar to the local function and \( \psi \) operator.

Different types of generalized open sets like semi-open[6], preopen[7], semi-peropen[2], \( \alpha \)-open[12] already are there in literature and these generalized sets have a common property which is closed under arbitrary union. Mashhour et al[8] put all
of the sets in a pocket and defined a generalized space which is supra topological space. In this space we have introduced ideal and defined two set operators, μ-local function and $\psi_\mu$ operator. Further we have discussed the properties of these two operators. Finally we have introduced μ-codense ideal, μ-compatible ideal and $\psi_\mu$-C set with the help of $\psi_\pi$ operator and discussed the properties of such notions.

2. Preliminaries

In this section we have discussed some definitions and results which are relevant of this paper.

**Definition 2.1.**[5] A nonempty collection I of subsets of X is called an ideal on X if:
(i). $A \in I$ and $B \subseteq A$ implies $B \in I$ (heredity);
(ii). $A \in I$ and $B \in I$ implies $A \cup B \in I$ (finite additivity).

**Definition 2.2.**[8] A subfamily $\mu$ of the power set $\wp(X)$ of a nonempty set X is called a supra topology on X if $\mu$ satisfies the following conditions:
1. $\mu$ contains $\emptyset$ and X,
2. $\mu$ is closed under the arbitrary union.
The pair $(X, \mu)$ is called a supra topological space.

In this respect, the member of $\mu$ is called supra open set in $(X, \mu)$. The complement of supra open set is called supra closed set.

**Definition 2.3.**[13] Let $(X, \mu)$ be a supra topological space and $A \subseteq X$. Then supra interior and supra closure of A in $(X, \mu)$ defined as $\bigcup\{U : U \subseteq A, \ U \in \mu\}$ and $\bigcap\{F : A \subseteq F, \ X - F \in \mu\}$ respectively.

The supra interior and supra closure of A in $(X, \mu)$ are denoted as $\text{Int}^\mu(A)$ and $\text{Cl}^\mu(A)$[13] respectively.

From definition, $\text{Int}^\mu(A)$ is a supra open set and $\text{Cl}^\mu(A)$ is a supra closed set.

**Definition 2.4.** Let $(X, \mu)$ be a supra topological space and $M \subseteq X$. Then M is said to a supra neighbourhood of a point $x$ of X if for some supra open set $U \in \mu, x \in U \subseteq M$.

The properties of $\text{Int}^\mu(A)$ and $\text{Cl}^\mu(A)$ have been discussed here which are relevant in this paper.

**Theorem 2.1.** Let $(X, \mu)$ be a supra topological space and $A \subseteq X$. Then
(i). $\text{Int}^\mu(A) \subseteq A$.
(ii). $A \in \mu$ if and only if $\text{Int}^\mu(A) = A$.
(iii). $\text{Cl}^\mu(A) \supseteq A$.
(iv). A is a supra closed set if and only if $\text{Cl}^\mu(A) = A$.
(v). $x \in \text{Cl}^\mu(A)$ if and only if every supra open set $U_x$ containing $x, U_x \cap A \neq \emptyset$.

Proof.(i). Proof is obvious from the definition of supra interior.
(ii). Since arbitrary union of supra open sets is again a supra open set, then proof is obvious.

(iii). Proof is obvious from the definition of supra closure.

(iv). If $A$ is a supra closed set, then smallest supra closed set containing $A$ is $A$. Hence $\text{Cl}_\mu(A) = A$.

(v). Let $x \in \text{Cl}_\mu(A)$. If possible suppose that $U_x \cap A = \emptyset$, where $U_x$ is a supra open set containing $x$. Then $A \subset (X - U_x)$ and $X - U_x$ is a supra closed set containing $A$. Therefore $x \in (X - U_x)$, a contradiction. Conversely supposed that $x \notin \text{Cl}_\mu(A)$, for every supra open set $U_x$ containing $x$. If possible suppose that $x \notin \text{Cl}_\mu(A)$, then $x \in X - \text{Cl}_\mu(A)$. Then there is a $U_x \in \mu$ such that $U_x \subset (X - \text{Cl}_\mu(A))$, i.e., $U_x \subset (X - \text{Cl}_\mu(A)) \subset (X - A)$. Hence $U_x \cap A = \emptyset$, a contradiction. So $x \in \text{Cl}_\mu(A)$.

Kuratowski in [5] has shown that $\text{int}A = X - \text{cl}(X - A)$ in topological space where ‘int’ and ‘cl’ denote the interior and closure operator in topological space. Following are the similar result in supra topological space $(X, \mu)$.

**Theorem 2.2.** Let $(X, \mu)$ be a supra topological space and $A \subset X$. Then $\text{Int}_\mu(A) = X - \text{Cl}_\mu(X - A)$.

**Proof.** Let $x \in \text{Int}_\mu(A)$. Then there is $U \in \mu$, such that $x \in U \subset A$. Hence $x \notin X - U$, i.e., $x \notin \text{Cl}_\mu(X - U)$, since $X - U$ is a supra closed set. So $x \notin \text{Cl}_\mu(X - A)$ (from Definition 2.3., $\text{Cl}_\mu(X - A) \subset \text{Cl}_\mu(X - U)$) and hence $x \in X - \text{Cl}_\mu(X - A)$.

Conversely suppose that $x \in X - \text{Cl}_\mu(X - A)$. So $x \notin \text{Cl}_\mu(X - A)$, then there is a supra open set $U_x$ containing $x$, such that $U_x \cap (X - A) = \emptyset$. So $U_x \subset A$. Therefore $x \in \text{Int}_\mu(A)$. Hence the result.

3. $(\cdot)^\mu$ operator

In [3] and [4] Hamlett and Jankovic have considered the local function in ideal topological space and they have obtained a new topology. In this section we shall introduce similar type of local function in supra topological space. Before starting the discussion we shall consider the following concept.

A supra topological space $(X, \mu)$ with an ideal $I$ on $X$ is called an ideal supra topological space and denoted as $(X, \mu, I)$.

At first we define following:

**Definition 3.1.** Let $(X, \mu, I)$ be an ideal supra topological space. A set operator $(\cdot)^\mu: \wp(X) \to \wp(X)$, is called the $\mu$-local function of $I$ on $X$ with respect to $\mu$, is defined as: $(A)^\mu(I, \mu) = \{ x \in X: U \cap A \notin I, \text{ for every } U \in \mu(x) \}$, where $\mu(x) = \{ U \in \mu: x \in U \}$.

This is simply called $\mu$-local function and simply denoted as $A^\mu$.

We have discussed the properties of $\mu$-local function in following theorem:
Theorem 3.1. Let \((X, \mu, I)\) be an ideal supra topological space, and let \(A, B, A_1, A_2, \ldots, A_i, \ldots\) be subsets of \(X\). Then

(i) \(\phi^\mu = \phi\).
(ii) \(A \subseteq B\) implies \(A^\mu \subseteq B^\mu\).
(iii) for another ideal \(J \supseteq I\) on \(X\), \(A^\mu(J) \subseteq A^\mu(I)\).
(iv) \(A^\mu \subseteq \text{Cl}^\mu(A)\).
(v) \(A^\mu\) is a supra closed set.
(vi) \((A^\mu)^\mu \subseteq A^\mu\).
(vii) \(A^\mu \cup B^\mu \subseteq (A \cup B)^\mu\).
(viii) \(\cap_i A_i^\mu \subseteq (\cap_i A_i)^\mu\).
(ix) \((A \cap B)^\mu \subseteq A^\mu \cap B^\mu\).
(x) for \(V \in \mu, (V \cap (V \cap A)^\mu) \subseteq V \cap A^\mu\).
(xi) for \(I \in \mu\), \((A \cup I)^\mu = A^\mu = (A - I)^\mu\).

Proof. (i). Proof is obvious from the definition of \(\mu\)-local function.
(ii). Let \(x \in A^\mu\). Then for every \(U \in \mu(x), U \cap A \notin I\). Since \(U \cap A \subseteq U \cap B\), then \(U \cap B \notin I\). This implies that \(x \in B^\mu\).
(iii). Let \(x \in A^\mu(J)\). Then for every \(U \in \mu(x), U \cap A \notin J\). This implies that \(U \cap A \notin I\), hence \(A^\mu(J) \subseteq A^\mu(I)\).
(iv). Let \(x \in A^\mu\). Then for every \(U \in \mu(x), U \cap A \notin I\). This implies that \(U \cap A \neq \phi\).
Hence \(x \in \text{Cl}^\mu(A)\).
(v). From definition of supra neighbourhood, each supra neighbourhood \(M\) of \(x\) contains a \(U \in \mu(x)\). Now if \(A \cap M \notin I\) then for \(A \cap U \subseteq A \cap M\), \(A \cap U \notin I\). It follows that \(X - A^\mu\) is the union of supra open sets. We know that the arbitrary union of supra open sets is a supra open set. So \(X - A^\mu\) is a supra open set and hence \(A^\mu\) is a supra closed set.
(vi). From (iv), \((A^\mu)^\mu \subseteq \text{Cl}^\mu(A^\mu) = A^\mu\), since \(A^\mu\) is a supra closed set.
(vii). We know that \(A \subseteq (A \cup B)\) and \(B \subseteq (A \cup B)\). Then from (ii), \(A^\mu \subseteq (A \cup B)^\mu\) and \(B^\mu \subseteq (A \cup B)^\mu\). Hence \(A^\mu \cup B^\mu \subseteq (A \cup B)^\mu\).
(viii). Proof is obvious from (vii).
(ix). We know that \(A \cap B \subseteq A\) and \(A \cap B \subseteq B\), then from (ii), \((A \cap B)^\mu \subseteq A^\mu\) and \((A \cap B)^\mu \subseteq B^\mu\). Hence \((A \cap B)^\mu \subseteq A^\mu \cap B^\mu\).
(x). Since \(V \cap A \subseteq A\), then \((V \cap A)^\mu \subseteq A^\mu\). So \(V \cap (V \cap A)^\mu \subseteq V \cap A^\mu\).
(xi). Since \(A \subseteq (A \cup I)\), then
\[A^\mu \subseteq (A \cup I)^\mu\] \[\text{-------------------(i).}\]
Let \(x \in (A \cup I)^\mu\). Then for every \(U \in \mu(x), U \cap (A \cup I) \notin I\). This implies that \(U \cap A \notin I\) (If possible suppose that \(U \cap A \in I\). Again \(U \cap I \in I\) implies \(U \cap I \notin I\) and hence \(U \cap (A \cup I) \notin I\;\text{, a contradiction}\). Hence \(x \in A^\mu\) and
\((A \cup I)^\mu \subseteq A^\mu\] \[\text{-------------------(ii).}\]
From (i) and (ii) we have
\[A^\mu = (A \cup I)^\mu\] \[\text{-------------------(ii).}\]
Since \((A - I) \subset A\), then
\[(A - I)^\mu \subset A^\mu \] \text{-------------------(iv).}

For reverse inclusion, let \(x \in A^\mu\). We claim that \(x \in (A - I)^\mu\), if not, then there is a \(U \in \mu(x)\), \(U \cap (A - I) \in I\). Given that \(I \in I\), then \(U \cup (U \cap (A - I)) \in I\). This implies that 
\(I \cup (U \cap A) \in I\). So, \(U \cap A \in I\), a contradiction to the fact that \(x \in A^\mu\). Hence
\[A^\mu \subset (A - I)^\mu \] \text{-------------------(v).}

From (iv) and (v) we have
\[A^\mu = (A - I)^\mu \] \text{-------------------(vi).}

Following example shows that \((A \cup B)^\mu = (A \cup B)^\mu\) does not hold in general.

**Example 3.1.** Let \(X = \{a, b, c, d\}\), \(\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, I = \{\emptyset, \{c\}\}\). Then supra open sets containing ‘a’ are: \(X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\); supra open sets containing ‘b’ are: \(X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\); supra open sets containing ‘c’ are: \(X, \{c\}, \{a, c\}, \{b, c\}, \{a, c, d\}, \{b, c, d\}\); supra open sets containing ‘d’ are: \(X, \{d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}\). Consider \(A = \{a, c\}\) and \(B = \{b, c\}\), then \(A^\mu = \{a\}\) and \(B^\mu = \{b\}\). Now \((A \cup B)^\mu = \{a, b, c\}^\mu = \{a, b, c, d\}\). Hence \((A \cup B)^\mu \neq (A \cup B)^\mu\).

Here \(A^\mu \cup B^\mu \neq (A \cup B)^\mu\), so we are not able to define a closure operator with the help of supra local function.

### 4. \(\psi_\mu\) operator

In this section we shall introduce another set operator \(\psi_\mu\) in \((X, \mu, I)\). This operator is, as like similar of \(\psi\) operator [10],[3] in ideal topological space.

**Definition 4.1.** Let \((X, \mu, I)\) be an ideal supra topological space. An operator \(\psi_\mu : \wp(X) \rightarrow \mu\) is defined as follows for every \(A \in \wp(X)\), \(\psi_\mu(A) = \{x \in X: \text{there exists a } U \in \mu(x) \text{ such that } U - A \in I\}\).

We observe that \(\psi_\mu(A) = X - (X - A)^\mu\).

The behaviors of the operator \(\psi_\mu\) has been discussed in the following theorem:

**Theorem 4.1.** Let \((X, \mu, I)\) be an ideal supra topological space.

(i). If \(A \subset X\), then \(\psi_\mu(A) \supset \text{Int}^I(A)\).

(ii). If \(A \subset X\), then \(\psi_\mu(A)\) is supra open.

(iii). If \(A \subset B\), then \(\psi_\mu(A) \subset \psi_\mu(B)\).

(iv). If \(A, B \in \wp(X)\), then \(\psi_\mu(A) \cup \psi_\mu(B) \subset \psi_\mu(A \cup B)\).

(v). If \(A, B \in \wp(X)\), then \(\psi_\mu(A \cap B) \subset \psi_\mu(A) \cap \psi_\mu(B)\).

(vi). If \(U \in \mu\), then \(U \subset \psi_\mu(U)\).

(vii). If \(A \subset X\), then \(\psi_\mu(A) \subset \psi_\mu(\psi_\mu(A))\).
If \( A \subseteq X \), then \( \psi_\mu(A) = \psi_\mu(\psi_\mu(A)) \) if and only if \( (X - A)^{\mu_*} = ((X - A)^{\mu})^{\mu} \).

If \( A \in I \), then \( \psi_\mu(A - I) = \psi_\mu(A) \).

If \( A \subseteq X \), \( I \in I \), then \( \psi_\mu(A \cup I) = \psi_\mu(A) \).

If \( (A - B) \cup (B - A) \in I \), then \( \psi_\mu(A) = \psi_\mu(B) \).

Proof.

(i). From definition of \( \psi_\mu \) operator, \( \psi_\mu(A) = X - (X - A)^{\mu} \). Then \( \psi_\mu(A) = X - (X - A)^{\mu} \supseteq X - \text{Cl}_\mu(X - A) \) from Theorem 3.1.(iv). Hence \( \psi_\mu(A) \supseteq \text{Int}_\mu(A) \) (using Theorem 2.2.).

(ii). Since \( (X - A)^{\mu} \) is a supra closed set (from Theorem 3.1(v)), then \( X - (X - A)^{\mu} \) is a supra open set. Hence \( \psi_\mu(A) \) is supra open.

(iii). Given that \( A \subseteq B \), then \( (X - A) \supseteq (X - B) \). Then from Theorem 3.1(ii), \( (X - A)^{\mu} \supseteq (X - B)^{\mu} \) and hence \( \psi_\mu(A) \subseteq \psi_\mu(B) \).

(iv). Proof is obvious from above property.

(v). Since \( A \cap B \subseteq A \) and \( A \cap B \subseteq B \), then from (iii), \( \psi_\mu(A \cap B) \subseteq \psi_\mu(A) \cap \psi_\mu(B) \).

(vi). Let \( U \in \mu \). Then \( (X - U) \) is a supra closed set and hence \( \text{Cl}_\mu(X - U) = (X - U) \). This implies that \( (X - U)^{\mu} \subset \text{Cl}_\mu(X - U) = (X - U) \). Hence \( U \subset X - (X - U)^{\mu} \), so \( U \subset \psi_\mu(U) \).

(vii). From (iii), \( \psi_\mu(A) \in \mu \). Again from (vi), \( \psi_\mu(A) \subset \psi_\mu(\psi_\mu(A)) \).

(viii). Let \( \psi_\mu(A) = \psi_\mu(\psi_\mu(A)) \). Then \( X - (X - A)^{\mu} = \psi_\mu(\psi_\mu(X - (X - A)^{\mu})) = X - ((X - (X - A)^{\mu})^{\mu}) = X - (X - A)^{\mu} \). This implies that \( (X - A)^{\mu} = ((X - A)^{\mu})^{\mu} \).

Conversely suppose that \( (X - A)^{\mu} = ((X - A)^{\mu})^{\mu} \) hold. Then \( X - (X - A)^{\mu} = X - ((X - A)^{\mu})^{\mu} \) and \( X - (X - A)^{\mu} = X - (X - (X - A)^{\mu})^{\mu} = X - (X - \psi_\mu(A))^{\mu} \).

Hence \( \psi_\mu(A) = \psi_\mu(\psi_\mu(A)) \).

(ix). We know that \( \psi_\mu(A) = X - (X - A)^{\mu} = X - X^{\mu} \) (from Theorem 3.1.(xi)).

(x). We know that \( X - (X - (A - I))^{\mu} = X - (X - (X - A))^{\mu} = X - (X - A)^{\mu} \) (from Theorem 3.1.(xi)). So \( \psi_\mu(A - I) = \psi_\mu(A) \).

(xi). We know that \( X - (X - (A \cup I))^{\mu} = X - ((X - A) \cup I)^{\mu} = X - (X - A)^{\mu} \) (using the Theorem 3.1.(xi)). Thus \( \psi_\mu(A \cup I) = \psi_\mu(A) \).

(xii). Given that \( (A - B) \cup (B - A) \in I \), and let \( A - B = I_1 \), \( B - A = I_2 \). We observe that \( I_1 \) and \( I_2 \in I \) by heredity. Also observe that \( B = (A - I_1) \cup I_2 \). Thus \( \psi_\mu(A) = \psi_\mu(A - I_1) = \psi_\mu((A - I_1) \cup I_2) = \psi_\mu(B) \).

We know that \( U \subset \psi_\mu(U) \), for \( U \in \mu \). But we give an example of a set \( A \) which is not supra open set but satisfies \( A \subseteq \psi_\mu(A) \).

**Example 4.1.** Let \( X = \{a, b, c, d\} \), \( \mu = \{ \phi, X, \{a\}, \{a,c,d\}, \{b,c,d\} \} \), \( I = \{ \phi, \{c\} \} \). Then for \( A = \{a,b,d\} \), \( \psi_\mu(A) = X - \{c\}^{\mu} = X - \phi = X \). Here \( A \subseteq \psi_\mu(A) \), but \( A \) is not a supra open set.

In the following example we shall show that \( \psi_\mu(A \cap B) = \psi_\mu(A) \cap \psi_\mu(B) \) does not hold in general.
**Example 4.2.** Consider the Example 3.1. Here we consider \( A = \{b,d\} \) and \( B = \{a,d\} \), then \( \psi_\mu(A) = X - \{a,c\}^\mu = X - \{a\} = \{b,c,d\} \) and \( \psi_\mu(B) = X - \{b,c\}^\mu = X - \{b\} = \{a,c,d\} \). Now \( \psi_\mu(\{d\}) = X - \{a,b,c\}^\mu = X - \{a,b,c,d\} = \phi \).

Here we are not able to define an interior operator with the help of \( \psi_\mu \) operator because \( \psi_\mu(A \cap B) \neq \psi_\mu(A) \cap \psi_\mu(B) \) in general.

---

### 5. \( \mu \)-codense Ideal

The study of ideal got new dimension when codense ideal \([4]\) has been incorporated in ideal topological space. In this section we introduce similar concept in ideal supra topological space.

**Definition 5.1.** An ideal \( I \) in a space \((X, \mu, I)\) is called \( \mu \)-codense ideal if \( \mu \cap I = \{\phi\} \).

Following theorems are related to \( \mu \)-codense ideal.

**Theorem 5.1.** Let \((X, \mu, I)\) be an ideal supra topological space and \( I \) is \( \mu \)-codense with \( \mu \). Then \( X = X^{\mu} \).

**Proof.**

It is obvious that \( X^{\mu} \subseteq X \). For converse, suppose \( x \in X \) but \( x \notin X^{\mu} \). Then there exists \( U_x \in \mu(x) \) such that \( U_x \cap X \in I \). That is \( U_x \in I \), a contradiction to the fact that \( \mu \cap I = \{\phi\} \). Hence \( X = X^{\mu} \).

**Theorem 5.2.** Let \((X, \mu, I)\) be an ideal supra topological space. Then following conditions are equivalent:

(i) \( \mu \cap I = \{\phi\} \).

(ii) \( \psi_\mu(\phi) = \phi \).

(iii) if \( i \in I \), then \( \psi_\mu(i) = \phi \).

**Proof.**

(i) \( \Rightarrow \) (ii). Given that \( \mu \cap I = \{\phi\} \), then \( \psi_\mu(\phi) = X - (X - \phi)^\mu = X - X^{\mu} = \phi \) (by Theorem 5.1).

(ii) \( \Rightarrow \) (iii). \( \psi_\mu(i) = X - (X - i)^\mu = X - X^{\mu} \) (by Theorem 3.1.(xi)) = \( X - X^{\mu} = \phi \) (by Theorem 5.1).

(iii) \( \Rightarrow \) (i). Suppose that \( A \in \mu \cap I \), then \( A \in I \) and by (iii), \( \psi_\mu(A) = \phi \). Again \( A \in \mu \), then by Theorem 4.1.(vi) we have \( A \subseteq \psi_\mu(A) = \phi \). Hence \( \mu \cap I = \{\phi\} \).

---

### 6. \( \mu \)-compatible Ideal

In this section we shall discuss a special type of ideal and its various properties. This special type of ideal is:

**Definition 6.1.** Let \((X, \mu, I)\) be an ideal supra topological space. We say the \( \mu \)-structure \( \mu \) is \( \mu \)-compatible with the ideal \( I \), denoted \( \mu \sim_\pi I \), if the following holds
for every $A \subseteq X$, if for every $x \in A$ there exists $U \in \mu(x)$ such that $U \cap A \in I$, then $A \in I$.

**Theorem 6.1.** Let $(X, \mu, I)$ be an ideal supra topological space. Then $\mu \sim I$ if and only if $\psi_\mu(A) - A \in I$ for every $A \subseteq X$.

**Proof.** Suppose $\mu \sim I$. Observe that $x \in \psi_\mu(A) - A$ if and only if $x \notin A$ and $x \notin (X - A)^\mu$ if and only if $x \notin A$ and there exists $U_x \in \mu(x)$ such that $U_x - A \in I$ if and only if there exists $U_x \in \mu(x)$ such that $x \in U_x - A \in I$. Now, for each $x \in \psi_\mu(A) - A$ and $U_x \in \mu(x)$, $U_x \cap (\psi_\mu(A) - A) \in I$ by heredity and hence $\psi_\mu(A) - A \in I$, since $\mu \sim I$.

Conversely suppose that the condition holds. Let $A \subseteq X$ and assume that for each $x \in A$ there exists $U_x \in \mu(x)$ such that $U_x \cap A \in I$.

From above Theorem we get following corollary:

**Corollary 6.1.** Let $(X, \mu, I)$ be an ideal supra topological space with $\mu \sim I$. Then $\psi_\mu(A) \subseteq \psi_\mu(\psi_\mu(A))$ for every $A \subseteq X$.

**Proof.** We know that $\psi_\mu(A) \subseteq \psi_\mu(\psi_\mu(A))$.

Newcomb in [11] has defined $A = B[\text{mod } I]$ if $(A - B) \cup (B - A) \in I$. Further he discussed various properties of $A = B[\text{mod } I]$.

Here we observe that if $A = B[\text{mod } I]$, then $\psi_\mu(A) = \psi_\mu(B)$.

Now we define Baire set in $(X, \mu, I)$.

**Definition 6.2.** Let $(X, \mu, I)$ be an ideal supra topological space. A subset $A$ of $X$ is called a Baire set with respect to $\mu$ and $I$, denoted $A \in \mathcal{B}_r(X, \mu, I)$, if there exists a supra open set $U \in \mu$ such that $A \subseteq U \subseteq A \cup I$.

**Theorem 6.2.** Let $(X, \mu, I)$ be an ideal supra topological space with $\mu \sim I$. If $U, V \in \mu$ and $\psi_\mu(U) = \psi_\mu(V)$, then $U = V[\text{mod } I]$.

**Proof.** Since $U \in \mu$, we have $U \subseteq \psi_\mu(U)$ and hence $U - V \subseteq \psi_\mu(U) - V = \psi_\mu(V)$ and $V \in I$ by Theorem 6.1. Similarly $V - U \in I$ and hence $U = V[\text{mod } I]$.

It is obvious that $A = B[\text{mod } I]$ is an equivalence relation. In this respect following theorem is remarkable:

**Theorem 6.3.** Let $(X, \mu, I)$ be an ideal supra topological space with $\mu \sim I$. If $A, B \in \mathcal{B}(X, \mu, I)$, and $\psi_\mu(A) = \psi_\mu(B)$, then $A = B[\text{mod } I]$.

**Proof.** Let $U, V \in \mu$ such that $A = U[\text{mod } I]$ and $B = V[\text{mod } I]$. Now $\psi_\mu(A) = \psi_\mu(B)$ and $\psi_\mu(B) = \psi_\mu(V)$ by Theorem 4.1.(xii). Since $\psi_\mu(A) = \psi_\mu(U)$ implies that $\psi_\mu(U) = \psi_\mu(V)$, hence $U = V[\text{mod } I]$ by Theorem 6.2. Hence $A = B[\text{mod } I]$ by transitivity.

**Theorem 6.4.** Let $(X, \mu, I)$ be an ideal supra topological space.

(i) If $B \in \mathcal{B}(X, \mu, I) - I$, then there exists $A \in \mu - \{\phi\}$ such that $B = A[\text{mod } I]$. 
(ii). Let \( \mu \cap I = \{ \phi \} \), then \( B \in B_r(X, \mu, I) - I \) if and only if there exist \( A \in \mu - \{ \phi \} \) such that \( B = A[\mod I] \).

Proof. Let \( B \in B_r(X, \mu, I) - I \). Then \( B \in B_r(X, \mu, I) \). Now if there does not exist \( A \in \mu - \{ \phi \} \) such that \( B = A[\mod I] \), we have \( B = \phi[\mod I] \). This implies that \( B \in I \) which is a contradiction.

(ii). Here we prove converse part only. Let \( A \in \mu - \{ \phi \} \) such that \( B = A[\mod I] \).

Then \( A = (B - J) \cup I \), where \( J = B - A \), \( I = A - B \in I \). If \( B \in I \), then \( A \in I \) by heredity and additivity, which contradict to \( \mu \cap I = \phi \).

7. \( \psi_\mu \)-C sets

Modak and Bandyopadhyay in [9] have introduced a generalized set with the help of \( \psi \)-operator in ideal topological space \((X, \mu, I)\). In this section we shall introduce a set with the help of \( \psi_\mu \) in \((X, \mu, I)\) space. Further we shall discuss the properties of this type of sets.

**Definition** 7.1. Let \((X, \mu, I)\) be an ideal supra topological space. A subset \( A \) of \( X \) is called a \( \psi_\mu \)-C set if \( A \subseteq Cl_\mu(\psi_\mu(A)) \).

The collection of all \( \psi_\mu \)-C sets in \((X, \mu, I)\) is denoted by \( \psi_\mu(X, \mu) \).

**Theorem** 7.1. Let \((X, \mu, I)\) be an ideal supra topological space. If \( A \in \mu \), then \( A \in \psi_\mu(X, \mu) \).

Proof. From Theorem 4.1.(vi) it follows that \( \mu \subseteq \psi_\mu(X, \mu) \).

Now we give an example which shows that the reverse inclusion is not true.

**Example** 7.1. Consider the Example 4.1. and we get \( A \in \psi_\mu(X, \mu) \) but \( A \notin \mu \).

We give an example which shows that any supra closed in \((X, \mu, I)\) may not be a \( \psi_\mu \)-C set.

In the following example, by \( C(\mu) \) we denote the family of all supra closed sets in \((X, \mu)\).

**Example** 7.2. We consider the Example 3.1. Here \( A = \{ d \} \in C(\mu) \). Then \( \psi_\mu(A) = X - \{ a, b, c \} \sim = X - X = \phi \). Therefore \( A \in C(\mu) \) but \( A \notin \psi_\mu(X, \mu) \).

**Theorem** 7.2. Let \( \{ A_\alpha : \alpha \in \Delta \} \) be a collection of nonempty \( \psi_\mu \)-C sets in an ideal supra topological space \((X, \mu, I)\), then \( \cup_{\alpha \in \Delta} A_\alpha \in \psi_\mu(X, \mu) \).

Proof. For each \( \alpha \in \Delta \), \( A_\alpha \subseteq Cl_\mu(\psi_\mu(A_\alpha)) \subseteq Cl_\mu(\psi_\mu(\cup_{\alpha \in \Delta} A_\alpha)) \). This implies that \( \cup_{\alpha \in \Delta} A_\alpha \subseteq Cl_\mu(\psi_\mu(\cup_{\alpha \in \Delta} A_\alpha)) \). Thus \( \cup_{\alpha \in \Delta} A_\alpha \in \psi_\mu(X, \mu) \).

The following example shows that the intersection of two \( \psi_\mu \)-C sets in \((X, \mu, I)\) may not be a \( \psi_\mu \)-C set.

**Example** 7.3. Consider the Example 4.2. Here we consider \( A = \{ b, d \} \) and \( B = \{ a, d \} \), then \( \psi_\mu(A) = X - \{ a, c \} \sim = X - \{ a \} = \{ b, c, d \} \) and \( \psi_\mu(B) = X - \{ b, c \} \sim = X - \{ b \} = \{ a, c, d \} \). So \( A, B \in \psi_\mu(X, \mu) \), but \( \psi_\mu(\{ d \}) = X - \{ a, b, c \} \sim = X - \{ a, b, c, d \} = \phi \), and \( \{ d \} \notin \psi_\mu(X, \mu) \).
References


Received: July, 2011