On Semi Generalized b-Continuous Maps, Semi Generalized b-Closed Maps in Topological Space

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Abstract

In this paper we introduce a new class of semi generalized b-continuous mapping (briefly sgb-continuous), semi generalized b-closed (briefly sgn-closed) map and studied some of its basic properties. We obtain some characterizations of such functions. Moreover we define approximately semi generalized b-continuous (briefly ap-sgb-continuous) and approximately semi generalized b-closed (briefly ap-sgb-closed) and open maps. Further we study certain properties of such functions.

Mathematics Subject Classification: 54C05, 54C10

1. INTRODUCTION

In 1996, Andrijevic [1] introduced one such new version called b-open sets. Caldas and S. Jafari [6] introduced on some applications of b-open sets in topological spaces. Levin [14] introduced the concept of generalized closed sets and studied their properties. A subset A of a space X is g-closed if and only if cl(A) ⊆ G whenever A ⊆ G and G is open. Hence every closed set is g-closed set. The union and intersection of two g-closed set is g-closed set. By considering the concept of g-closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematicians. Dunham and Levin [8] further studied some properties of g-closed sets. Arya and Nour [3]. Bhattachary and Lahiri [5], Levin [12][13], and H. Maki, R.Devi and K. Balachandran [18] introduced and investigated gs-open sets, sg-open sets, generalized open sets, semi-open sets and g α-closed sets which are some of the weak forms of open (closed) sets and the complements of these sets are called same types of closed (open) sets respectively.
Ahmad et al., [2] studied and investigated on generalized b-closed (abbreviated gb-closed) sets. Hence every b-closed set is gb-closed set. Iyappan and Nagaveni [10] introduced a new version called semi generalized b-closed (briefly sgb-closed) set and also discovered the properties, the similarities and the difference of this set with other form of closed sets in topological spaces. The purpose of this paper is to introduce a new version of continuous functions called semi-generalized b-continuous (briefly sgb-continuous) and a new classes of maps namely semi generalized b-closed maps and semi generalized b-open maps. Moreover we introduce the concepts of approximately semi generalized b-continuous (briefly ap-sgb-continuous) and approximately sgb-closed and approximately sgb-open maps of topological spaces and we investigated the properties of all such transformations.

The aim of this paper is to continue the study of sgb-continuous with the relation of other continuous functions in section 3. In section 4, we introduce and discussed the relation of other closed and open maps with sgb-closed and open maps. Finally in section 5 we introduce a new class of approximately sgb-continuous, approximately sgb-closed and open maps and discussed the relation with other continuous, closed and open maps in topological spaces.

2. PRELIMINARIES

In this section we begin by recalling some definition and properties.
Let \((X, \tau)\) be a topological space and \(A\) be a subset. The closure of \(A\) and interior of \(A\) are denoted by \(\text{Cl}(A)\) and \(\text{Int}(A)\) respectively. We recall some generalized closed sets.

Definition 2.1 : A subset of a topological space \((X, \tau)\) is called

   \[ \text{bCl}(A) \subseteq G \]
   whenever \(A \subseteq G\) and \(G\) is semi-open.
2. b-closed set [12] if
   \[ \text{Int}(\text{Cl}(A)) \cap \text{Cl}(\text{Int}(A)) \subseteq A \]
3. \(\alpha\)-closed [9] if
   \[ A \subseteq \text{Int}(\text{Cl}(\text{Int}(A))) \]
4. A generalized b-closed (briefly gb-closed) [10] if \(\text{bCl}(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is open.
5. A weakly-generalized closed set (briefly wg-closed) [12] if \(\text{Cl}(\text{Int} A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is open.
6. A semi-weakly generalized closed set (briefly swg-closed) [12] if \(\text{Cl}(\text{Int} A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is semi-open.
7. An \(\alpha\)-generalized closed set (briefly \(\alpha\) g-closed set) [9] if \(\alpha \text{Cl}(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is open.
8. A generalized semi-closed set (briefly gs-closed) [5] if \(\text{sCl}(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is open.
9. A semi-generalized closed set (briefly sg-closed) [6] if \(\text{sCl}(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is semi open.
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A \text{ g}^\ast \text{-closed set} [13] if \text{Cl}(A) \subseteq G whenever A \subseteq G and G is semi-open set.

A generalized \alpha \text{-closed set} (briefly g\alpha \text{-closed}) [6] if \alpha \text{ Cl}(A) \subseteq G whenever A \subseteq G and G is \alpha \text{-open set.}

Definition [9] 2.2 : A map \( f : X \rightarrow Y \) is called b-continuous if \( f^{-1}(F) \) is b-closed set in X for each closed set F in Y.

Definition [7] 2.3 : A map \( f : X \rightarrow Y \) is called contra-continuous if \( f^{-1}(F) \) is closed set in X for each open set F in Y.

Definition [20] 2.4 : A map \( f : X \rightarrow Y \) is called contra b-continuous if \( f^{-1}(F) \) is b-closed set in X for each closed set F in Y.

Definition [9] 2.5 : A map \( f : X \rightarrow Y \) is called b-closed if \( f(F) \) is b-closed set in Y for every b-closed subset F in X.

Definition [20] 2.6 : A map \( f : X \rightarrow Y \) is called contra b-closed if \( f(F) \) is b-open set in Y for every closed subset F in X.

Definition [14] 2.7 : A map \( f : X \rightarrow Y \) is called generalized closed (g-closed) set if each closed set F of X, \( f(F) \) is g-closed set in Y.

Definition [12] 2.8 : A map \( f : X \rightarrow Y \) is called semi-closed if each closed set F of X, \( f(F) \) is semi closed in Y.

Definition [16] 2.9 : A map \( f : X \rightarrow Y \) is called \alpha \text{-closed} if each closed set F of X, \( f(F) \) is \alpha \text{-closed set in Y.}

Definition [15] 2.10 : A map \( f : X \rightarrow Y \) is called regular-closed if for each closed subset F of X, \( f(F) \) is regular-closed in Y.

Definition [19] 2.11 : A map \( f : X \rightarrow Y \) is called weakly-generalized-closed (wg-closed) (resp. semi-weakly generalized-closed(briefly swg-closed)) if for each closed subset F of X, \( f(F) \) is wg-closed (resp. swg-closed) in Y.

Definition [22] 2.12 : A map \( f : X \rightarrow Y \) is called semi generalized-closed (briefly sg-closed)(resp. generalized semi-closed(briefly gs-closed)) if for each closed subset F of X, \( f(F) \) is sg-closed in Y.

Definition [17] 2.13 : A map \( f : X \rightarrow Y \) is called g-closed if \( f(F) \) is g-closed set in Y for each closed set F in X.

Definition [16] 2.14 : A map \( f : X \rightarrow Y \) is called \alpha \text{-continuous} if \( f^{-1}(F) \) is \alpha \text{-open} in X for every open set V in Y.

Definition [4] 2.15 : A map \( f : X \rightarrow Y \) is called g-continuous if \( f^{-1}(F) \) is g-closed in X for every closed set F in Y.

Definition [21] 2.16 : A map \( f : X \rightarrow Y \) is called perfectly-continuous if \( f^{-1}(F) \) is both open and closed in X for every open set F in Y.

Definition [19] 2.17 : A map \( f : X \rightarrow Y \) is called weakly generalized continuous(briefly wg-continuous)(resp. Semi weakly generalized continuous
(briefly swg-continuous)) if $f^{-1}(F)$ is wg-closed (resp. swg-closed) in $X$ for every closed set $F$ in $Y$.

Definition [12] 2.18 : A map $f : X \to Y$ is called semi-continuous if $f^{-1}(F)$ is semi-closed in $X$ for every closed subset $F$ in $Y$.

Definition [4] 2.19 : A map $f : X \to Y$ is called semi generalized-continuous (briefly sg-continuous) (resp. generalized semi-continuous (briefly gs-continuous)) if $f^{-1}(F)$ is sg-closed (resp. gs-closed) in $X$ for every closed subset $F$ in $Y$.

Remark [10][11] 2.20: The following relation has been proved for the sgb-closed set.

Remark [10][11] 2.22: If a subset $A$ of $X$ is b-closed or swg-closed or g$\alpha$-closed or sg-closed or g$\hat{\alpha}$-closed, then it is sgb-closed.

Where $A \rightarrow B$ (resp. $A \leftrightarrow B$) represent $A$ implies $B$ (resp. $A$ and $B$ independent).

Remark [10][11] 2.22: If a subset $A$ of $X$ is b-closed or swg-closed or g$\alpha$-closed or sg-closed or g$\hat{\alpha}$-closed, then it is sgb-closed.
In this section we introduce sgb-continuous mappings and study some of their properties.

Definition 3.1: Let $X$ and $Y$ be topological spaces, a map $f : X \to Y$ said to be semi generalized b-continuous (abbreviated as sgb-continuous) if the inverse image of every open set in $Y$ is sgb-open in $X$.

Theorem 3.2: If a map $f : X \to Y$ from a topological space $X$ into a topological space $Y$ is continuous then it is sgb-continuous but not conversely.

Proof: Let $V$ be an open set in $Y$. Since $f$ is continuous, $f^{-1}(V)$ is open in $X$. As open set is sgb – open, $f^{-1}(V)$ is sgb-open in $X$. Therefore $f$ is sgb-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = \{a, b, c\} \Rightarrow Y$, with topologies $\tau = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$, and $\sigma = \{Y, \emptyset, \{a, b\}\}$. Let $f : X \to Y$ be defined by $f(a) = b$, $f(b) = c$, and $f(c) = a$ then $f$ is sgb-continuous but not continuous as the inverse image of the open set $\{a, b\}$ in $Y$ is $\{a, c\}$ not in open $X$.

Theorem 3.4: A map $f : X \to Y$ is sgb-continuous if and only if inverse image of every closed set in $Y$ is sgb-closed in $X$.

Proof: Let us assume that $f$ is sgb-continuous, let $F$ be closed set in $Y$ then $F^c$ is open in $Y$. Since $f$ is sgb-continuous, $f^{-1}(F^c)$ is sgb-open in $X$. But $f^{-1}(F^c) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is sgb-closed in $X$. Conversely, assume that inverse image of every closed set in $Y$ is sgb-closed in $X$. Let $V$ be an open set in $Y$ and $V^c$ be closed in $Y$. By hypothesis, $f^{-1}(V^c) = X - f^{-1}(V)$ is sgb-closed in $X$ and so, $f^{-1}(V)$ is sgb-open in $X$. Thus, $f$ is sgb-continuous.

Theorem 3.5: Let $f : X \to Y$ be a topological space $X$ into a topological space $Y$

(a) The following statements are equivalent
   (i) $f$ is sgb-continuous
   (ii) The inverse image of every open in $Y$ is sgb-open in $X$

(b) If $f : X \to Y$ is sgb – continuous, then $f(\overline{A}) \subseteq f(A)$ for every subset $A$ of $X$. [here $\overline{A}$ is the closure of $A$ as defined by Dunham [8]; For a subset $A$ of a space $(X, \tau)$, $\overline{A}$ denotes the closure of $A$ in $(X, \tau)$]

(c) The following statements are equivalent
   (i) For each point $x \in X$ and each open set $V$ containing $f(x)$ there exists a sgb-open set $U$ containing $x$ such that $f(U) \subseteq V$
   (ii) For every subset $A$ of $X$, $f(\overline{A}) \subseteq f(A)$ holds
   (iii) The map $f : (X, \tau^*) \to (Y, \sigma)$ from a topological space $(X, \tau^*)$ defined by Dunham [8] into a topological space $(Y, \sigma)$ is continuous.

Proof: (a) The equivalence is proved by definition.

(b) Let $f : X \to Y$ is sgb – continuous, since $A \subseteq f^{-1}(\overline{A})$, it is
obtained that $f(\text{cl}^*(A)) \subset f(A)$ by using assumption, but converse need not true as seen from the following example.

Example 3.6: Let $x = \{a, b, c\} = y$ with topologies $\tau = \{X, \varnothing, \{a\}\}$ and $\sigma = \{Y, \varnothing, \{a\}\}$. Let $f: X \to Y$ be a map defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then for every subset $A$, $f(\text{cl}^*(A)) \subset f(A)$ holds but $f$ is not sgb-continuous.

(c) (i) ($\Rightarrow$) Let $y \in f(\text{cl}^*(A))$ and let $V$ be any open neighbourhood of $y$, then there exist a point $x \in X$ and a sgb-open set $U$ such that $f(x) = y$, $x \in U$, $x \in \text{cl}^*(A)$ and $f(U) \subset V$. Since $x \in \text{cl}^*(A)$, $U \cap A \neq \varnothing$ holds and hence $f(A) \cap V \neq \varnothing$. Therefore, we have $y = f(x) \in \overline{f(A)}$.

(ii) ($\Rightarrow$) Let $x \in X$ and $V$ be any open set containing $f(x)$. Let $A = f^{-1}(V)$, then $x \in A$. Since $f(\text{cl}^*(A)) \subset \overline{f(A)} \subset V^\tau$, it is show that $\text{cl}^*(A) = A$. Then, since $x \notin \text{cl}^*(A)$, there exist a g-open set $U$ containing $x$, such that $U \cap A = \varnothing$ and hence $f(U) \subset f(Ac) \subset V$.

(iii) ($\Rightarrow$) by Dunhan [8] the closure of $A$ in $(X, \tau^*)$ coincides with the set $\text{cl}^*(A)$. Therefore, the equivalence is proved by standard arguments.

Theorem 3.7: If a map $f: X \to Y$ is perfectly continuous then it is sgb-continuous.

Proof: Let $f: X \to Y$ is perfectly continuous, $V$ be an open set in $Y$. Then $f^{-1}(V)$ is both open and closed in $X$, and hence $f^{-1}(V)$ is sgb-open in $X$. Thus $f$ is sgb-continuous.

Remarks 3.8: The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \varnothing, \{a\}\}$ and $\sigma = \{Y, \varnothing, \{a\}\}$. Let $f: X \to Y$ be defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ then $f$ is sgb-continuous but not perfectly continuous as the inverse image of the open set $\{a, b\}$ in $Y$ is $\{a, b\}$ not both open and closed in $X$.

Theorem 3.10: If a map $f: X \to Y$ is $\alpha$-continuous then it is sgb-continuous.

Proof: Let $f: X \to Y$ in $\alpha$-continuous, $V$ be a closed set in $Y$, then $f^{-1}(V)$ is $\alpha$-open in $X$. As $\alpha$-open implies b-open and also implies that sgb-open, so $f^{-1}(V)$ is sgb-open in $X$. Thus $f$ is sgb-continuous.

Remark 3.11: The converse of the above theorem need not be true as seen from the following example.

Example 3.12: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \varnothing, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \varnothing, \{a\}\}$. Let $f: X \to Y$ be defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ then $f$ is sgb-continuous but not $\alpha$-continuous as the inverse image of the open set $\{a, b\}$ in $Y$ is $\{a, b\}$ not $\alpha$-open in $X$.

Theorem 3.13: If a map $f: X \to Y$ is semi-continuous then it is sgb-continuous.

Proof: Let $f: X \to Y$ is semi-continuous, $V$ be a closed set in $Y$, then $f^{-1}(V)$ is semi closed in $X$, thus $f^{-1}(V)$ is sg-closed in $X$. Hence $f$ is sgb-continuous.

Remark 3.14: The converse of the above theorem need not be true as seen from the following example.
Example 3.15: Let \( x = \{a, b, c\} = y \) with topologies \( \tau = \{X, \varnothing, \{b, c\}\} \) and \( \sigma = \{Y, \varnothing, \{a\}\} \). Let \( f: X \to Y \) be defined by \( f(a) = b, f(b) = c \) and \( f(c) = a \) then \( f \) is sgb-continuous but not semi-continuous as the inverse image of the closed set \( \{b, c\} \) in \( y \) is \( \{a, b\} \) not semi-closed in \( X \).

Theorem 3.16: If a map \( f: X \to Y \) is swg-continuous then it is sgb-continuous.

Proof: Let \( f: X \to Y \) be swg-continuous, \( V \) be an open set in \( Y \). Since \( f \) is swg-continuous, \( f^{-1}(V) \) is swg-open in \( X \) then \( f^{-1}(V) \) is sgb-open in \( X \). Thus \( f \) is sgb-continuous.

Remark 3.17: The converse of the above theorem need not be true as seen from the following example.

Example 3.18: Let \( X = \{a, b, c\} = Y \) with topologies \( \tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\} \) and \( \sigma = \{Y, \varnothing, \{c\}\} \). Let \( f: X \to Y \) be defined by \( f(a) = b, f(b) = c \) and \( f(c) = a \) then \( f \) is sgb-continuous but not swg-continuous as the inverse image of the open set \( \{a\} \) in \( Y \) is \( \{b, c\} \) not swg-open in \( X \).

Theorem 3.19: If a map \( f: X \to Y \) is b-continuous then it is sgb-continuous.

Proof: Let \( f: X \to Y \) be b-continuous, \( V \) be an open set in \( Y \). Since \( f \) is b-continuous, \( f^{-1}(V) \) is b-open in \( X \), then \( f^{-1}(V) \) is sgb-open in \( X \). Thus \( f \) is sgb-continuous.

Remark 3.20: The converse of the above theorem need not be true as seen from the following example.

Example 3.21: Let \( X = \{a, b, c\} = Y \) with topologies \( \tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\} \) and \( \sigma = \{Y, \varnothing, \{c\}\} \). Let \( f: X \to Y \) be defined by \( f(a) = a, f(b) = c \) and \( f(c) = b \). Then \( f \) is sgb-continuous but not b-continuous as the inverse image of the open set \( \{a\} \) in \( Y \) is \( \{a\} \) not b-open in \( X \).

Theorem 3.22: If a map \( f: X \to Y \) is \( g\alpha \)-continuous then it is sgb-continuous.

Proof: Let \( f: X \to Y \) be \( g\alpha \)-continuous, \( V \) be an open set in \( Y \), then \( f^{-1}(V) \) is \( g\alpha \)-open in \( X \), thus \( f^{-1}(V) \) is sgb-open in \( X \), therefore \( f \) is sgb-continuous.

Remark 3.23: The converse of the above theorem need not be true as seen from the following example.

Example 3.24: Let \( X = \{a, b, c\} = Y \) with topologies \( \tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\} \) and \( \sigma = \{Y, \varnothing, \{c\}\} \). Let \( f: X \to Y \) be defined by \( f(a) = a, f(b) = a \) and \( f(c) = b \) then \( f \) is sgb-continuous but not \( g\alpha \)-continuous as the inverse image of the open set \( \{c\} \) in \( Y \) is \( \{a\} \) not \( g\alpha \)-open in \( X \).

Theorem 3.25: If a map \( f: X \to Y \) is \( \hat{g} \)-continuous then it is sgb-continuous.

Proof: Let \( f: X \to Y \) be \( \hat{g} \)-continuous, \( V \) be an open set in \( Y \), then \( f^{-1}(V) \) is \( \hat{g} \)-open in \( X \), thus \( f^{-1}(V) \) is sgb-open in \( X \) therefore \( f \) is sgb-continuous.

Remark 3.26: The converse of the above theorem need not be true as seen from the following example.

Example 3.27: Let \( X = \{a, b, c\} = Y \) with topologies \( \tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{b, c\}\} \) and \( \sigma = \{Y, \varnothing, \{c\}\} \). Let \( f: X \to Y \) be defined by \( f(a) = b, f(b) = a \) and \( f(c) = c \). Then \( f \) is sgb-continuous but not \( \hat{g} \)-continuous as the inverse image of the open set \( \{c\} \) in \( Y \) is \( \{c\} \) not \( \hat{g} \)-open in \( X \).
Remark 3.28: The following examples shows that sgb-continuous and wg-continuous maps are independent.

Example 3.29: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}\}$. Let $f : X \to Y$ be defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then $f$ is sgb-continuous but not wg-continuous.

Example 3.30: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{b\}\}$. Let $f : X \to Y$ be defined by $f(a) = c$, $f(b) = c$ and $f(c) = b$. Then $f$ is wg-continuous but not sgb-continuous.

Remark 3.31: The following examples shows that sgb-continuous and generalized semi-continuous maps are independent.

Example 3.32: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{c\}\}$. Let $f : X \to Y$ be defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then $f$ is sgb-continuous but not gsemi-continuous.

Example 3.33: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{c\}\}$. Let $f : X \to Y$ be defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then $f$ is gsemi-continuous but not sgb-continuous.

Remark 3.34: The following example show that sgb-continuous and $\alpha g$-continuous, gs-continuous, gsp-continuous, gp-continuous and gb-continuous are independent.

Example 3.35: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{c\}\}$. Let $f : X \to Y$ be defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then $f$ is sgb-continuous but not $\alpha g$-continuous, gs-continuous, gsp-continuous gp-continuous and gb-continuous.

Example 3.36: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{a\}\}$ and $\sigma = \{Y, \emptyset, \{b\}\}$. Let $f : X \to Y$ be defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then $f$ is $\alpha g$-continuous, gs-continuous, gsp-continuous gp-continuous and gb-continuous but not sgb-continuous.
Remark 3.37: From the above discussion we get the following diagram.

![Diagram](image)

Where $A \rightarrow B$ (resp. $A \leftrightarrow B$) represent $A$ implies $B$ (resp. $A$ and $B$ independent).

Theorem 3.38: Let $X$ and $Z$ be any topological space and $Y$ be a $T_{sgb}$-space. Then the composition $g \circ f : X \rightarrow Z$ of the sgb-continuous maps $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ is also sgb-continuous.
Proof: If follows from definition of $T_{sgb}$-space which is defined as every $sgb$-closed set is semi-closed.

Remark: The following example shows that the above proposition need not be true if $Y$ is not $T_{sgb}$

Example: Let $X=Y=Z=\{a, b, c\}$ with topological spaces $\tau = \{X, \emptyset, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b, c\}\}$ and $\eta = \{Z, \emptyset, \{a, \}, \{b\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ be defined by $f(a) = b, f(b) = b$ and $f(c) = a$. Then $f$ and $g$ are $sgb$-continuous but $g \circ f$ is not $sgb$-continuous as the inverse image of the closed set $\{b, c\}$ in $Z$ is $\{a, b\}$ not $sgb$ closed set in $X$.

4. SEMI GENERALIZED b-CLOSED MAPS

In this section we introduce $sgb$-closed and $sgb$-open maps and study some of their properties.

Definition 4.1: A map $f: X \rightarrow Y$ is said to be $sgb$ – open if the image of every open set in $X$ is $sgb$ – open in $Y$.

Definition 4.2: A map $f: X \rightarrow Y$ is said to be $sgb$ closed if the image of every closed set in $X$ is $sgb$-closed in $Y$.

Theorem 4.3: Every open map is $sgb$ – open but not conversely.

Proof: Let $f: X \rightarrow Y$ is an open map and $V$ be an open set in $X$. Then $f(V)$ is open and hence $sgb$ – open in $Y$. Thus $f$ is $sgb$ – open.

Remark 4.4: The converse of the above theorem need not be true as seen from the following example.

Example 4.5: let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{b\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{c\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ be defined by $f(a) = c, f(b) = b$ and $f(c) = a$. Then $f$ is $sgb$ – open but not open as the image of the open set $\{b\}$ in $X$ is $\{b\}$ not open set in $Y$.

Theorem 4.6: Every closed map is $sgb$ - closed but not conversely.

Proof: Let $f: X \rightarrow Y$ be closed map and $V$ be a closed set in $X$. Then $f(V)$ is closed and hence $sgb$ – closed in $Y$. Thus $f$ is $sgb$ – closed.

Remark 4.7: The converse of the above theorem need not be true as seen from the following example.

Example 4.8: let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \emptyset, \{a, c\}, \{b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{c\}\}$. Let $f: X \rightarrow Y$ be defined by $f(a) = c, f(b) = b$ and $f(c) = a$. Then $f$ is $sgb$- closed but not closed as the image of the closed set $\{b\}$ in $X$ is $\{b\}$ not closed set in $Y$.

Theorem 4.9: A map $f: X \rightarrow Y$ in $sgb$-closed if and only if subset $S$ of $Y$ and for each open set $U$ containing $f^{-1}(S)$ there is a $sgb$ – open set $V$ of $Y$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Suppose $f$ is $sgb$- closed. Let $S$ be a subset of $Y$ and $U$ be an open set of $X$ such that $f^{-1}(S) \subseteq U$ then $V = Y - f(X - U)$ is a $sgb$ – open set containing $S$ such that $f^{-1}(V) \subseteq U$.
Conversely, suppose that $F$ is a closed of $X$. Then $f^{-1}(Y - f(F)) \subseteq X - F$ and $X - F$ is open. By hypothesis, there is a sgb – open set $V$ of $Y$ such that $Y - f(F) \subseteq V$ and $f^{-1}(V) \subseteq X - F$ Therefore, $F \subseteq X - f^{-1}(V)$. Hence $Y - V \subseteq f(F) \subseteq f(X - f^{-1}(V)) \subseteq Y - V$ which implies $f(F) = Y - V$. Since $Y - V$ is sgb- closed, $f(F)$ is sgb- closed and thus $f$ is sgb- closed map.

Theorem 4.10 : If a map $f : X \to Y$ is continuous and sgb- closed and $A$ is sgb-closed set of $X$, $f(A)$ is sgb-closed in $Y$.

Proof : Let $f(A) \subseteq O$, when $O$ is an open set of $Y$. Since $f$ is continuous $f^{-1}(O)$ is an open set containing $A$. Hence $bcl (A) \subseteq f^{-1}(O)$ as $A$ is sgb-closed set. Since $f$ is sgb-closed, $f(bcl (A))$ is sgb- closed set contained in the open set $O$, which implies $bcl (f(cl (A))) \subseteq O$ and hence $bcl (f(A)) \subseteq O$. So, $f(A)$ is sgb-closed in $Y$.

Theorem 4.11 : If a map $f : X \to Y$ is sgb- closed and continuous and $A$ is sgb-closed set of $X$, then $fA : A \to Y$ is continuous and sgb- closed.

Proof : Let $F$ be a closed set of $A$. Then $F$ is sgb- closed of $X$. From Theorem 4.10 it follow that $fA (F) = f(F)$ is sgb- closed set if $Y$. Hence $fA$ is sgb- closed and also continuous.

Theorem 4.12 : If a map $f : X \to Y$ is sgb- closed and continuous and $A$ is sgb-closed set of $X$, then $fA : A \to Y$ is sgb- closed.

Proof : Let $H$ be a closed set in $X$, then $f(H)$ is sgb- closed and $(g \circ f)(H) = g(f(H))$ is sgb- closed as $g$ is sgb- closed. Thus $g \circ f$ is sgb- closed.

Theorem 4.13 : If a map $f : X \to Y$ is $\alpha$- open then it is sgb – open.

Proof : Let $H$ be an open set in $X$ then $f(H)$ is $\alpha$-open in $Y$, as $f$ is $\alpha$-open and hence if is sgb – open in $Y$. Therefore $f$ is sgb – open.

Theorem 4.14 : If a map $f : X \to Y$ is $b$ – open then if it is sgb – open

Proof : Let $V$ be an open set in $X$ then $f(V)$ is $b$ – open in $Y$ as $f$ is $b$ – open and hence it is sgb – open in $Y$. Therefore $f$ is sgb – open.

Theorem 4.15 : If a map $f : X \to Y$ is $sg$ – open then if it is sgb – open.

Proof : Let $V$ be a open set in $X$ then $f(V)$ is $sg$ – open in $Y$ as $f$ is $sg$ – open and hence it is sgb – open in $Y$. Therefore $f$ is sgb – open.

Corollary 4.16: If a map $f : X \to Y$ is $swg$ – open, $\tilde{g}$ – open, $g\alpha$ - open then it is sgb – open.

Theorem 4.17: If a map $f : X \to Y$ is regular closed and a map $g : Y \to Z$ is sgb-closed then $g \circ f : X \to Z$ is sgb-closed.

Proof : Let $f : X \to Y$ is regular closed, let $V$ be a closed set in $X$ then $f(V)$ is regular closed set in $Y$ and $(g \circ f)(V) = g(f(V))$ is sgb- closed as $g$ is sgb- closed. Thus $(g \circ f)$ is sgb- closed.

Corollary 4.18: If $f : X \to Y$ is $b$-continuous and closed and $A$ is $b$-closed set in $X$ then $f(A)$ is sgb-closed in $Y$.

Theorem 4.19 : If a map $f : X \to Y$ is $b$-closed and a map $g : Y \to Z$ is sgb-closed then $g \circ f : X \to Z$ is sgb-closed.

Proof : Let $H$ be a $b$-closed in $X$ then $f(H)$ is $b$-closed in $Y$ and $(g \circ f)(H) = g(f(H))$ is sgb- closed as $g$ is sgb-closed. Thus $(g \circ f)$ is sgb- closed.

Theorem 4.20: If a map $f : X \to Y$ is closed and a map $g : Y \to Z$ is $b$-closed
then \( g \circ f : X \rightarrow Z \) is sgb- closed.

**Proof:** Let \( H \) be a closed set in \( X \) then \( f(H) \) is closed set in \( Y \) and \((g \circ f)(H) = g(f(H)) \) is b-closed in \( Y \) as \( g \) is b – closed, and hence \( g(f(H)) \) is sgb- closed in \( Y \).

**Corollary 4.21:** If a map \( f : X \rightarrow Y \) is closed and a map \( g : Y \rightarrow Z \) swg – closed or \( \mathfrak{g} \) – closed or \( g \alpha \)-closed or sg – closed then \( g \circ f : X \rightarrow Z \) is sgb- closed.

**Theorem 4.21:** If \( f : X \rightarrow Y \) is a continuous, sgb- closed map from a normal space \( X \) onto a space \( Y \), then \( Y \) is normal.

**Proof:** Let \( A, B \) be disjoint closed sets of \( Y \). Then \( f^{-1}(A) \) and \( f^{-1}(B) \) are disjoint closed sets of \( X \). Since \( X \) is normal there are disjoint open sets \( U, V \) in \( X \) such that \( f^{-1}(A) \subseteq U \) and \( f^{-1}(B) \subseteq V \). Since \( f \) is sgb- closed, by theorem 4.9 there are open sets \( G, H \) in \( Y \) such that \( A \subseteq G, B \subseteq H, f^{-1}(G) \subseteq U \) and \( f^{-1}(H) \subseteq V \). Since \( U, V \) are disjoint, int\( G \) and int\( H \) are disjoint open sets. Since \( G \) is sgb-open \( A \) is closed and \( A \subseteq G, A \subseteq \text{int}G \). Similarly \( B \subseteq \text{int}H \). Hence \( Y \) is normal.

**Theorem 4.22:** If a map \( f : X \rightarrow Y \) is an open, continuous, sgb- closed surjection where \( X \) is regular, then \( Y \) is regular.

**Proof:** Let \( U \) be an open set containing a point of \( X \). Such that \( f(x) = p \). Since \( X \) is regular and \( f \) is continuous, there is an open set \( V \) such that \( x \in V \subseteq \text{cl} V \subseteq f^{-1}(U) \). Hence \( p \in f(V) \subseteq f(\text{cl}(V)) \subseteq U \). Since \( f \) is sgb-closed, \( f(\text{cl}(V)) \) is sgb- closed set contained in the open set \( U \). It follows that bcl \((f(\text{cl}(V))) \subseteq U \) and hence \( p \in f(V) \subseteq \text{cl} f(V) \subseteq U \) and \( f(V) \) is open. Since \( f \) is open. Hence \( Y \) is regular.

**Theorem 4.23:** If \( f : X \rightarrow Y \) is sgb- closed and \( A = f^{-1}(B) \) for some closed set \( B \) of \( Y \) then \( f_A : A \rightarrow Y \) is sgb- closed.

**Proof:** Let \( F \) be a closed set in \( A \). Then there is a closed set \( H \) in \( X \) such that \( F = A \cap H \). Then \( f_A(F) = f(A \cap H) = f(H) \cap f(B) \). Since \( f \) is sgb- closed \( f(H) \) is sgb- closed in \( Y \). So, \( f(H) \cap B \) is sgb- closed in \( Y \). Since the intersection of a sgb- closed and closed set is a sgb- closed set. Hence \( f_A \) is sgb- closed.

### 5. APPROXIMATELY SEMI GENERALIZED b-CONTINUOUS FUNCTIONS

In this section we introduce a new class of approximately semi generalized b-continuous (briefly ap-sgb-continuous), approximately semi generalized b-closed and open maps (briefly ap-sgb-closed map, ap-sgb-open map) and discussed with this properties with some other continuous functions.

**Definition 5.1:** A map \( f : X \rightarrow Y \) is said to be approximately sgb-continuous (briefly ap – sgb – continuous) if \( \text{bcl}(F) \subseteq f^{-1}(U) \) whenever \( U \) is a semi open subset of \( Y \), and \( F \) is a sgb- closed subset of \( X \) such that \( F \subseteq f^{-1}(U) \).
Definition 5.2: A map $f : X \to Y$ is said to be approximately - sgb- closed (briefly ap – sgb- closed) if $f(F) \subseteq \text{bint}(V)$ whenever $V$ is a sgb – open subset of $Y$, $F$ is a semi – closed subset of $X$ and $f(F) \subseteq V$.

Definition 5.3: A map $f : X \to Y$ is said to be approximately sgb – open (briefly ap – sgb – open) if $\text{bcl}(F) \subseteq f(U)$ whenever $U$ is semi-open subset of $X$, $F$ is a sgb- closed subset of $Y$ and $F \subseteq f(U)$.

Theorem 5.4: Let $f : X \to Y$ be a function, then

(a) If $f$ is contra b – continuous, then $f$ is ap – sgb – continuous.
(b) If $f$ is contra b – closed, then $f$ is ap – sgb- closed.
(c) If $f$ is contra b – open, then $f$ is ap – b – open.

Proof:

(a) Let $F \subseteq f^{-1}(U)$, when $U$ in semi – open in $Y$ and $F$ is a sgb- closed subset of $X$. Therefore $\text{bcl}(F) \subseteq \text{bcl}(f^{-1}(U))$. Since $f$ is contra – b-continuous we have $\text{bcl}(f) \subseteq \text{bcl}(f^{-1}(U)) = f^{-1}(U)$. Thus $f$ is ap – sgb – continuous.

(b) Let $f(F) \subseteq V$, where $F$ is a semi closed sub set of $X$ and $V$ is sgb – open sub set of $Y$. Therefore $f(F) = \text{bint}(f(F)) \subseteq \text{bint}(V)$. Thus $f$ is ap – sgb- closed.

(c) Analogous to (a) & (b) making the obvious changes.

Example 5.5: Let $X = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a, b\}\}$ Then $\text{bcl}(X) = \{X, \varnothing, \{a\}, \{b\}\}$. Let $f : X \to X$ be the identity map. Then $f$ is ap – sgb-continuous but not contra – b-continuous as the inverse image of the open set $\{a, b\}$ in $Y$ is $\{a, b\}$ not b-closed in $Y$.

Example 5.6: Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{X, \varnothing, \{a\}\}$ and $\sigma = \{Y, \varnothing, \{a\}, \{a, b\}\}$ then $\text{bO}(X) = \{X, \varnothing, \{a\}, \{a, b\}\}$, $\text{sgbC}(X) = \{X, \varnothing, \{a\}, \{b\}\}$ and $\text{bO}(Y) = \{Y, \varnothing, \{a\}, \{a, b\}\}$, $\text{sgbC}(Y) = \{Y, \varnothing, \{b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map then $f$ is ap – sgb- closed but not contra – b-closed as the image of the closed set $\{b, c\}$ in $X$ is $\{bc\}$ not b – open in $Y$. Also the function in example 6.5 is ap – sgb – open, but not contra – b – open clearly, the following two diagram holds and note of its implications is reversible.

```
perfectly continuous -------> contra – b-continuous
Continuous                     app – sgb-continuous
perfectly closed-------------> contra – b-closed-------------> app – sgb- closed.
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