Intuitionistic Fuzzy Pre Semi Basically Disconnected Space

D. Amsaveni, M. K. Uma and E. Roja

Sri Sarada College for Women, Salem-636016
Tamilnadu, India
d_amsaveni@rediffmail.com

Abstract

In this paper, a new class of intuitionistic fuzzy topological spaces called intuitionistic fuzzy pre semi basically disconnected spaces is introduced. Tietze extension theorem for intuitionistic fuzzy pre semi basically disconnected spaces has been discussed as in [12] and several other properties are also discussed.

Keywords: Intuitionistic fuzzy pre semi disconnected spaces and lower (resp. upper) intuitionistic fuzzy pre semi continuous functions

1 Introduction

After the introduction of the concept of fuzzy sets by Zadeh[12], several researches were conducted on the generalizations of the notion of fuzzy set. The concept of ”Intuitionistic fuzzy sets” was first published by Atanassov[2] and many works by the same author and his colleagues appeared in the literature [3-5]. Later this concept was generalized to ”Intuitionistic L-fuzzy sets” by Atanassov and Stoeva[6]. An introduction to intuitionistic fuzzy topological space was introduced by Dogan Coker[9]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Coker(1997). The construction is based on the idea of intuitionistic fuzzy set developed by Atanassov(1983, 1986, Atanassov and Stoeva, 1983). In this paper a new class of intuitionistic fuzzy topological spaces namely, intuitionistic fuzzy pre semi basically disconnected spaces is introduced by using the concepts of fuzzy basically disconnected spaces[11].’Intuitionistic fuzzy pre semi closed sets’ was introduced by [1]. Tietze extension theorem for ordered intuitionistic fuzzy pre semi basically disconnected spaces has been discussed as in [12]. Some interesting properties and characterizations are studied.
2 Preliminaries

Throughout this paper let $X$ be a non empty set and $I = [0, 1]$.

Definition 2.1. [4] Let $X$ be a nonempty fixed set and $I$ is the closed interval $[0, 1]$. An intuitionistic fuzzy set (IFS for short) $A$ is an object having the form $A = \{ (x, \mu_A(x), \gamma_A(x)) : x \in X \}$ where the mapping $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) for each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Remark 2.1. [4] For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A \rangle$.

Definition 2.2. [9] Let $X$ be a non empty fixed set. Then $0_\sim = \{ (x, 0, 1) : x \in X \}$ and $1_\sim = \{ (x, 1, 0) : x \in X \}$.

Definition 2.3. [9] Let $X$ be a non empty fixed set. An intuitionistic fuzzy topology (IFT for short) on a non empty set $X$ is a family $\tau$ of intuitionistic fuzzy sets (IFSs for short) in $X$ satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(iii) $\bigcup G_i \in \tau$ for arbitrary family $\{ G_i \ | \ i \in J \} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFS for short) in $X$.

Definition 2.4. [9] Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in $X$. Then the fuzzy interior and fuzzy closure of $A$ are defined by

\[ \text{int}(A) = \bigcup \{ G \ | \ G \text{ is an IFOS in } X \text{ and } G \subseteq A \}; \]
\[ \text{cl}A = \bigcap \{ K \ | \ K \text{ is an IFCS in } X \text{ and } A \supseteq K \}. \]

Notation 2.1. An IFTS $(X, T)$ represent intuitionistic fuzzy topological spaces and for a subset $A$ of space $(X, T)$, IFcl$(A)$, IFint$(A)$, IFPScl$(A)$, IFPSint$(A)$ and $\overline{A}$ denote an intuitionistic fuzzy closure of $A$, intuitionistic fuzzy interior of $A$, intuitionistic fuzzy pre semi closure of $A$, intuitionistic fuzzy pre semi interior of $A$ and the complement of $A$ in $X$ respectively.

Definition 2.5. [14] A subset $A$ of an IFTS $(X, T)$ is called an IF semi pre open set if $A \subseteq \text{IFcl}(\text{IFint}(\text{IFcl}(A)))$ and an IF semi pre closed set if IFint(IFcl(IFint(A))) $\subseteq A$. 

Definition 2.6. [10] A subset $A$ of an IFTS $(X, T)$ is called an IF generalized closed (briefly IF g-closed) set if $IF cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is IF open set in $(X, T)$. The complement of IF g-closed set is called an IF g-open set.

Definition 2.7. [1] A subset $A$ of an IFTS $(X, T)$ is called an intuitionistic fuzzy pre semi closed (IF pre semi closed for short) set if $IF spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is IF g-open set in $(X, T)$.

Definition 2.8. [1] A subset $A$ of an IFTS $(X, T)$ is called an intuitionistic fuzzy pre semi open (IF pre semi open for short) set if $A$ is IF pre semi closed set.

Definition 2.9. [1] A function $f : (X, T) \rightarrow (Y, S)$ is called intuitionistic fuzzy pre semi continuous (IF pre semi continuous for short) if $f^{-1}(V)$ is an IF pre semi closed set of $(X, T)$ for every IF closed set $V$ of $(Y, S)$.

Definition 2.10. [7] Let $(X, T)$ be a fuzzy topological space and $\lambda$ be a fuzzy set in $(X, T)$. $\lambda$ is called a fuzzy $G_\delta$ if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$.

Definition 2.11. [7] Let $(X, T)$ be a fuzzy topological space and $\lambda$ be a fuzzy set in $(X, T)$. $\lambda$ is called a fuzzy $F_\sigma$ if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$.

Definition 2.12. [7] Let $(X, T)$ be any fuzzy topological space. $(X, T)$ is called fuzzy basically disconnected if the closure of every fuzzy open $F_\sigma$ set is fuzzy open.

3 Intuitionistic fuzzy pre semi basically disconnected Spaces

In this section a new class of space called intuitionistic fuzzy pre semi basically disconnected space is introduced. Some interesting properties and characterizations are also discussed.

Definition 3.1. Let $(X, T)$ be an intuitionistic fuzzy topological space (for short IFTS) and $A$ be an intuitionistic fuzzy set (for short IFS) in $(X, T)$. $A$ is called an intuitionistic fuzzy pre semi $G_\delta$ (for short IF pre semi $G_\delta$) if $A = \bigcap_{i=1}^{\infty} A_i$ where each $A_i$ is IF pre semi open.

Definition 3.2. Let $(X, T)$ be an intuitionistic fuzzy topological space (for short IFTS) and $A$ be an intuitionistic fuzzy set (for short IFS) in $(X, T)$. $A$ is called an intuitionistic fuzzy pre semi $F_\sigma$ (for short IF pre semi $F_\sigma$) if $A = \bigcup_{i=1}^{\infty} A_i$ where each $\overline{A_i}$ is IF pre semi open.
Note 3.1. (a) Let \((X, T)\) be an IFTS. An IFS \(A\) in \((X, T)\) which is both intuitionistic fuzzy pre semi open (for short, IF pre semi open) and \(IFF_{\sigma}\) is denoted by IF pre semi open \(F_{\sigma}\).

(b) Let \((X, T)\) be an IFTS. An IFS \(A\) in \((X, T)\) which is both intuitionistic fuzzy pre semi closed (for short, IF pre semi closed) and \(IFG_{\delta}\) is denoted by IF pre semi closed \(G_{\delta}\).

(c) An IFS \(A\) which is both IF pre semi open \(F_{\sigma}\) and IF pre semi closed \(G_{\delta}\) is denoted by intuitionistic fuzzy pre semi COGF (for short, IF pre semi COGF).

Definition 3.3. Let \((X, T)\) be an IFTS. Let \(A\) be any IF pre semi open \(F_{\sigma}\) in \((X, T)\). If IF pre semi closure of \(A\) is IF pre semi open \(F_{\sigma}\), then \((X, T)\) is said to be intuitionistic fuzzy pre semi basically disconnected (for short IF pre semi basically disconnected).

Proposition 3.1. For an IFTS \((X, T)\), the following statements are equivalent:

(a) \((X, T)\) is IF pre semi basically disconnected.

(b) For each IF pre semi closed \(G_{\delta}\) set \(A\), we have \(IFPS_{\text{int}}F_{\sigma}(A)\) is IF pre semi closed \(G_{\delta}\).

(c) For each IF pre semi open \(F_{\sigma}\) set \(A\), we have \(IFPS_{\text{cl}}G_{\delta}(IFPS_{\text{int}}F_{\sigma}(\overline{A})) = IFPS_{\text{cl}}G_{\delta}(A)\).

(d) For each pair of IF pre semi open \(F_{\sigma}\) sets \(A\) and \(B\) in \((X, T)\) with \(IFPS_{\text{cl}}G_{\delta}(A) = B\), we have \(IFPS_{\text{cl}}G_{\delta}(A) = IFPS_{\text{cl}}G_{\delta}(B)\).

Proposition 3.2. Let \((X, T)\) be an IFTS. Then \((X, T)\) is an IF pre semi basically disconnected space if and only if for any IF pre semi open \(F_{\sigma}\) set \(A\) and IF pre semi closed \(G_{\delta}\) set \(B\) such that \(A \subseteq B\), \(IFPS_{\text{cl}}G_{\delta}A \subseteq IFPS_{\text{int}}F_{\sigma}B\).

Notation 3.1. An IFS which is both IF pre semi open \(F_{\sigma}\) and IF pre semi closed \(G_{\delta}\) is called IF pre semi COGF set.

Remark 3.1. Let \((X, T)\) be an IF pre semi basically disconnected space. Let \(\{A_i, \overline{B}_i \mid i \in N\}\) be an collection such that \(A_i\)'s are IF pre semi open \(F_{\sigma}\) sets, \(B_i\)'s are IF pre semi closed \(G_{\delta}\) sets and let \(A, \overline{B}\) be IF pre semi COGF. If \(A_i \subseteq A \subseteq B_j\) and \(A_i \subseteq B \subseteq B_j\) for all \(i, j \in N\), then there exists a IF pre semi COGF set \(C\) such that \(IFPS_{\text{cl}}G_{\delta}(A_i) \subseteq C \subseteq IFPS_{\text{int}}F_{\sigma}(B_j)\) for all \(i, j \in N\).
Proposition 3.3. Let \((X, T)\) be an IF pre semi basically disconnected space. Let \((A_q)_{q \in Q}\) and \((A_q)_{q \in Q}\) be the monotone increasing collections of IF pre semi open \(F_{\sigma}\) sets and IF pre semi closed \(G_{\delta}\) sets of \((X, T)\) respectively and suppose that \(A_{q_1} \subseteq B_{q_2}\) whenever \(q_1 < q_2\) \((Q)\) is the set of rational numbers). Then there exists a monotone increasing collection \(\{C_q\}_{q \in Q}\) of IF pre semi COGF sets of \((X, T)\) such that \(IFPSclG_\delta(A_{q_1}) \subseteq C_{q_2}\) and \(C_{q_1} \subseteq IFPSintF_\sigma(B_{q_2})\) whenever \(q_1 < q_2\).

4 Properties and characterizations of intuitionistic fuzzy pre semi basically disconnected Spaces

In this section various properties and characterizations of intuitionistic fuzzy pre semi basically disconnected spaces are discussed.

Definition 4.1. Let \((X, T)\) be an IFTS. A function \(f : X \rightarrow R(I)\) is called lower(resp. upper)intuitionistic fuzzy pre semi continuous(for short IF pre semi continuous), if \(f^{-1}(R_i)(\text{resp. } f^{-1}(L_i))\) is an IF pre semi open \(F_{\sigma}\) set(resp. IF pre semi open \(F_{\sigma}/IF\) pre semi closed \(G_{\delta}\)) set for each \(t \in R\).

Proposition 4.1. Let \((X, T)\) be an IFTS and let \(A\) be an IFS in \(X\) and let \(f : X \rightarrow R(I)\) be such that

\[
f(x)(t) = \begin{cases} 
1 & \text{if } t < 0 \\
A(x) & \text{if } 0 \leq t \leq 1 \\
0 & \text{if } t > 1
\end{cases}
\]

for all \(x \in X\). Then \(f\) is lower(resp. upper)IF pre semi continuous function iff \(A\) is an IF pre semi open \(F_{\sigma}\) set(resp. IF pre semi open \(F_{\sigma}/IF\) pre semi closed \(G_{\delta}\)) set.

Definition 4.2. Let \((X, T)\) be an IFTS. The characterization function of \(IFSA\) in \(X\) is the function \(\chi_A : X \rightarrow I(L)\) defined by \(\chi_A(x) = (A(x)), (x \in X)\)

Proposition 4.2. Let \((X, T)\) be an IFTS and let \(A\) be an IFS in \(X\). Then \(\chi_A\) is lower(resp. upper)IF pre semi continuous function iff \(A\) is an IF pre semi open \(F_{\sigma}\) set(resp. IF pre semi open \(F_{\sigma}/IF\) pre semi closed \(G_{\delta}\)) set.

Definition 4.3. Let \((X, T)\) and \((Y, S)\) be IFTSs. A function \(f : (X, T) \rightarrow (Y, S)\) is called intuitionistic fuzzy strongly \(F_{\sigma}\) pre semi continuous(for short IF strongly \(F_{\sigma}\) pre semi continuous) if \(f^{-1}(A)\) is IF pre semi COGF in \((X, T)\) for every IF pre semi open \(F_{\sigma}\) set \((Y, S)\).

Proposition 4.3. Let \((X, T)\) be an IFTS. Then the following are equivalent

(a) \((X, T)\) is IF pre semi basically disconnected
(b) If \( g, h : X \to \mathbb{R}\(R), g \) is lower \( IF \) pre semi continuous, \( h \) is upper \( IF \) pre semi continuous and \( g \subseteq h \), then there exists an \( IF \) strongly \( F_\sigma \) pre semi continuous function, \( f : (X,T) \to \mathbb{R}\(R) \) such that \( g \subseteq f \subseteq h \).

(c) If \( \overline{A} \) and \( B \) are \( IF \) pre semi open \( F_\sigma \) sets such that \( B \subseteq A \), there exists an \( IF \) strongly \( F_\sigma \) pre semi continuous function \( f : (X,T) \to [0,1](I) \) such that \( B \subseteq \overline{\text{L}_1 f} \subseteq \text{R}_o f \subseteq A \).

5 Tietze extension theorem for \( IF \) pre semi basically disconnected Spaces

In this section Tietze extension theorem for \( IF \) pre semi basically disconnected spaces is studied.

**Proposition 5.1.** Let \((X,T)\) be an upper \( IF \) pre semi basically disconnected spaces and let \( A \subset X \) be such that \( \chi_A \) is \( IF \) pre semi open \( F_\sigma \) set in \((X,T)\). Let \( f : (A,T/A) \to [0,1](I) \) be an \( IF \) strongly \( F_\sigma \) pre semi continuous function. Then \( f \) has an \( IF \) strongly \( F_\sigma \) pre semi continuous extension over \((X,T)\).

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**References**


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