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Common Fixed Point Theorems of R -weakly Commuting Mappings in Fuzzy Metric Spaces

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Abstract

In this paper, we prove a common fixed point theorems using R -weak commutativity in fuzzy metric spaces.

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Keywords: Compatible mapping, reciprocal continuity, R -weakly commuting mapping, t -norm of Hadžić-type

1 Introduction

Fuzzy set theory has many applications in applied sciences such as neural network theory, stability theory, mathematical programming, modelling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc. Various authors such as Erceg [1], George and Veermani [2], Kaleva and Seikkala [6], Kramosil and Michálek [7] introduced the concept of fuzzy metric spaces in different ways. Here we are considering the fuzzy metric space in the sense of Kramosil and Michálek [7].

Definition 1.1. A binary operation Δ on $[0, 1]$ is a *t-norm* if it satisfies the following conditions:

- (i) Δ is associative and commutative,
- (ii) $\Delta(a, 1) = a$ for every $a \in [0, 1]$,
- (iii) $\Delta(ab) \leq \Delta(c, d)$, whenever $a \leq c$ and $b \leq d$.

Basics examples of *t-norm* are the Lukasiewicz *t-norm* Δ_L , $\Delta_L(a, b) = \max\{a + b - 1, 0\}$, the product *t-norm* Δ_P , $\Delta_P(a, b) = ab$ and the minimum *t-norm* Δ_M , $\Delta_M(a, b) = \min\{a, b\}$.

Definition 1.2. ([4]) Let Δ be a *t-norm* and $\Delta_n : [0, 1] \rightarrow [0, 1]$ ($n \in \mathbb{N}$) be defined by

$$\Delta_1(x) = \Delta(x, x), \quad \Delta_{n+1}(x) = \Delta(\Delta_n(x), x) \quad (n \in \mathbb{N}, x \in [0, 1]).$$

Then we say that the *t-norm* Δ is of *Hadžić-type* if the family $\{\Delta_n(x); n \in \mathbb{N}\}$ is equicontinuous at $x = 1$. The family $\{\Delta_n(x); n \in \mathbb{N}\}$ is said to be *equicontinuous* at $x = 1$ if for every $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that

$$x > 1 - \delta(\lambda) \quad \text{implies} \quad \Delta_n(x) > 1 - \lambda \quad (n \in \mathbb{N}).$$

A trivial example of *t-norm* of Hadžić-type is $\Delta = \Delta_M$.

Remark 1.3. ([4]) (1) If there exists a strictly increasing sequence $\{b_n\}_{n \in \mathbb{N}}$ in $[0, 1]$ such that $\lim_{n \rightarrow \infty} b_n = 1$ and $\Delta(b_n, b_n) = b_n$ for all $n \in \mathbb{N}$, then Δ is of Hadžić-type.

(2) If Δ is continuous and of Hadžić-type, then there exists a sequence $\{b_n\}_{n \in \mathbb{N}}$ as in (1).

Definition 1.4. ([4]) If Δ is a *t-norm* and $(x_1, x_2, x_3, \dots, x_n) \in [0, 1]^n$ ($n \in \mathbb{N}$), then $\Delta_{i=1}^n x_i$ is defined recurrently by 1 if $n = 1$ and $\Delta_{i=1}^n x_i = \Delta(\Delta_{i=1}^{n-1} x_i, x_n)$ for all $n \geq 2$. If $\{x_n\}_{n \in \mathbb{N}}$ is a sequence in $[0, 1]$, then $\Delta_{i=1}^\infty x_i$ is defined as $\lim_{n \rightarrow \infty} \Delta_{i=1}^n x_i$ (this limit always exists) and $\Delta_{i=1}^\infty x_i$ as $\Delta_{i=1}^\infty x_{n+i}$.

Definition 1.5. The 3-triple (X, M, Δ) is called a *fuzzy metric space* in the sense of Kramosil and Michálek if X is an arbitrary set, Δ is a continuous t -norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: For all $x, y, z \in X$ and $s, t > 0$

- (FM-1) $M(x, y, 0) = 0$,
- (FM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
- (FM-3) $M(x, y, t) = M(y, x, t)$,
- (FM-4) $M(x, z, t + s) \geq \Delta(M(x, y, t), M(y, z, s))$,
- (FM-5) $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous.

Note that $M(x, y, t)$ can be thought of as the degree of nearness between x and y with respect to t .

Remark 1.6. ([3]) In a fuzzy metric space (X, M, Δ) , $M(x, y, t)$ is non-decreasing with respect to t for all $x, y \in X$.

Definition 1.7. A sequence $\{x_n\}$ in a fuzzy metric space (X, M, Δ) is said to be

- (1) *convergent* to the limit x if $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$ for all $t > 0$.
- (2) *Cauchy sequence* in X if for every $\lambda \in (0, 1)$ and $t > 0$, there exists a positive integer N such that $M(x_n, x_m, t) > 1 - \lambda$ whenever $m, n \geq N$.
- (3) *complete* if every Cauchy sequence in X is convergent in X .

Fixed point theory in fuzzy metric spaces has been developing since the paper of Grabiec [3]. Subramanian [11] gave a generalization of Jungck's theorem ([5]) for commuting mapping in the setting of fuzzy metric spaces.

In 1994, Mishra et al. [8] generalised the notion of weakly commuting to compatible mappings in fuzzy metric spaces akin to the concept of compatible mapping in metric spaces (see [5]).

Definition 1.8. Let f and g be self-mappings on a fuzzy metric space (X, M, Δ) . Then a pair (f, g) is said to be *compatible* if $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = u$ for some $u \in X$ and for all $t > 0$.

In 1994, Pant [9] introduced the concept of R -weakly commuting maps in metric spaces. Later on, Vasuki [12] introduced the notion of R -weakly commuting mapping in fuzzy metric spaces and proved some common fixed point theorems for these mappings.

Definition 1.9. Let f and g be self-mappings on a fuzzy metric space (X, M, Δ) . Then a pair (f, g) is said to be

- (1) *weakly commuting* if $M(fgx, gfx, t) \geq M(fx, gx, t)$ for all $x \in X$ and $t > 0$.
- (2) *R -weakly commuting* if there exists $R > 0$ such that $M(fgx, gfx, t) \geq M(fx, gx, t/R)$ for all $x \in X$ and $t > 0$.

In 1999, Pant [10] introduced a new continuity condition, known as reciprocal continuity as follows.

Definition 1.10. Let f and g be self-mappings on a fuzzy metric space (X, M, Δ) . Then f and g are called *reciprocally continuous* if $\lim_{n \rightarrow \infty} f g x_n = f z$ and $\lim_{n \rightarrow \infty} g f x_n = g z$, whenever $\{x_n\}$ is a sequence such that $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = z$ for some $z \in X$.

If f and g are both continuous, then they are obviously reciprocally continuous, but the converse is need not be true.

Lemma 1.11. ([8]) *Let (X, M, Δ) be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.*

2 Main Results

Lemma 2.1. *Let (X, M, Δ) be a fuzzy metric space with continuous t -norm of Hadžić-type. Let A, B, S and T be self-mappings on X satisfying the following conditions:*

$$(C1) \quad A(X) \subset T(X), \quad B(X) \subset S(X),$$

(C2) *there exists $q \in (0, 1)$ such that*

$$M(Ax, By, qt) \geq \min\{M(Ty, By, t), M(Sx, Ax, t), M(Sx, By, (2 - \alpha)t), \\ M(Ty, Ax, \alpha t), M(Ty, Sx, t)\}$$

for every $x, y \in X$ and $t > 0$, where $\alpha \in (0, 2)$. Then the continuity of one of the mappings in compatible pairs (A, S) or (B, T) on (X, M, Δ) implies their reciprocal continuity.

Proof. First assume that A and S are compatible and S is continuous. We show that A and S are reciprocally continuous. Let $\{x_n\}$ be a sequence such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ for some $z \in X$. Since S is continuous, we have $\lim_{n \rightarrow \infty} S Ax_n = Sz$ and $\lim_{n \rightarrow \infty} S S x_n = Sz$ and since (A, S) is compatible, we have for all $t > 0$, $\lim_{n \rightarrow \infty} M(ASx_n, S Ax_n, t) = 1$ and hence $\lim_{n \rightarrow \infty} ASx_n = Sz$.

By (C1), for each n , there exists $\{y_n\}$ in X such that $ASx_n = Ty_n$. Thus, we have

$$\lim_{n \rightarrow \infty} S S x_n = \lim_{n \rightarrow \infty} S A x_n = \lim_{n \rightarrow \infty} A S x_n = Sz$$

and $\lim_{n \rightarrow \infty} Ty_n = Sz$.

Now we claim that $\lim_{n \rightarrow \infty} By_n = Sz$. Take $\alpha = 1$ in (C2), we have

$$\begin{aligned} M(ASx_n, By, qt) \\ \geq \min\{M(Ty_n, By_n, t), M(SSx_n, ASx_n, t), M(SSx_n, By_n, t), \\ M(Ty_n, ASx_n, t), M(Ty_n, SSx_n, t)\} \end{aligned}$$

Taking limit as $n \rightarrow \infty$ and using Lemma 1.11, we have $\lim_{n \rightarrow \infty} By_n = Sz$.

Next we claim that $Az = Sz$. Again take $\alpha = 1$ in (C2), we have

$$\begin{aligned} M(Az, By_n, qt) \geq \min\{M(Ty_n, By_n, t), M(Sz, Az, t), M(Sz, By_n, t), \\ M(Ty_n, Az, t), M(Ty_n, Sz, t)\}. \end{aligned}$$

Taking limit as $n \rightarrow \infty$, we have

$$\begin{aligned} M(Az, Sz, qt) &\geq \min\{M(Sz, Sz, t), M(Sz, Az, t), M(Sz, Sz, t), \\ &\quad M(Sz, Az, t), M(Sz, Sz, t)\} \\ &= \min\{1, M(Sz, Az, t), 1, M(Sz, Az, t), 1\} \\ &\geq M(Sz, Az, t). \end{aligned}$$

By Lemma 1.11, we have $Az = Sz$. Therefore, $\lim_{n \rightarrow \infty} SAx_n = Sz$ and $\lim_{n \rightarrow \infty} ASx_n = Sz = Az$. Hence, A and S are reciprocally continuous on X .

If the pair (B, T) is compatible and T is continuous. The proof is similar. This completes the proof. $\square$

Theorem 2.2. *Let (X, M, Δ) be a fuzzy metric space with continuous minimum t -norm of Hadžić-type. Let A, B, S and T be self-mappings on X satisfying (C1), (C2) and the following conditions:*

(C3) *If one of the mappings in compatible pair (A, S) or (B, T) is continuous,*

(C4) *the pair (A, S) and (B, T) are R -weakly commuting mappings.*

Assume that there exist $x_0, x_1 \in X$ such that for $y_1 = Ax_0 = Tx_1$, $y_2 = Bx_1 = Sx_2$ and $\mu \in (q, 1)$

$$\lim_{n \rightarrow \infty} \Delta_{i=n}^{\infty} M(y_1, y_2, 1/\mu^i) = 1.$$

Then A, B, S and T have a unique common fixed point in X .

Proof. Since $B(X) \subset S(X)$, there exist $x_1, x_2 \in X$ such that $Bx_1 = Sx_2$. Inductively, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ of X such that

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}, \quad y_{2n} = Sx_{2n} = Bx_{2n-1}$$

for $n = 1, 2, \dots$

Take $\alpha = 1 - \beta$ in (C2), since $\Delta(a, b) = \min\{a, b\}$, we have

$$\begin{aligned}
& M(y_{2n+1}, y_{2n+2}, qt) \\
&= M(Ax_{2n}, Bx_{2n+1}, qt) \\
&\geq \min\{M(Tx_{2n+1}, Bx_{2n+1}, t), M(Sx_{2n}, Ax_{2n}, t), M(Sx_{2n}, Bx_{2n+1}, (1 + \beta)t), \\
&\quad M(Tx_{2n+1}, Ax_{2n}, (1 - \beta)t), M(Tx_{2n+1}, Sx_{2n}, t)\} \\
&= \min\{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+2}, (1 + \beta)t), \\
&\quad M(y_{2n+1}, y_{2n+1}, (1 - \beta)t), M(y_{2n+1}, y_{2n}, t)\} \\
&\geq \min\{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), \\
&\quad \Delta(M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, \beta t)), 1, M(y_{2n+1}, y_{2n}, t)\} \\
&= \min\{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), 1, M(y_{2n+1}, y_{2n+2}, \beta t)\}.
\end{aligned}$$

Taking $\beta \rightarrow 1$, we have

$$M(y_{2n+1}, y_{2n+2}, qt) \geq \min\{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), 1\}.$$

By Remark 1.6,

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t).$$

Similarly, we have

$$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t).$$

In general, for all $t > 0$ and $n = 1, 2, 3, \dots$

$$M(y_n, y_{n+1}, qt) \geq M(y_{n-1}, y_n, t).$$

Thus for all $t > 0$ and $n = 1, 2, 3, \dots$

$$\begin{aligned}
M(y_n, y_{n+1}, qt) &\geq M(y_{n-1}, y_n, t) \\
&\geq M(y_{n-2}, y_{n-1}, t/q) \\
&\quad \dots \\
&\geq M(y_1, y_2, t/q^{n-2}).
\end{aligned}$$

Now, we show that $\{y_n\}$ is a Cauchy sequence in X .

Let $\sigma = \frac{q}{\mu}$. Since $0 < \sigma < 1$, the series $\sum_{i=1}^{\infty} \sigma^i$ is convergent and there exists $m_0 \in \mathbb{N}$ such that $\sum_{i=m_0}^{\infty} \sigma^i < 1$. Hence for every $m > m_0 + 1$ and $s \in \mathbb{N}$

$$t > t \sum_{i=m_0}^{\infty} \sigma^i > t \sum_{i=m-1}^{m+s-1} \sigma^i.$$

Now

$$\begin{aligned}
& M(y_{m+s+1}, y_m, t) \\
& \geq M\left(y_{m+s+1}, y_m, t \sum_{i=m-1}^{m+s-1} \sigma^i\right) \\
& \geq M(y_{m+s+1}, y_m, t\sigma^{m-1} + t\sigma^{m-1+1} + t\sigma^{m-1+2} + \dots + t\sigma^{m-1+s}) \\
& \geq M(y_{m+s+1}, y_m, t\sigma^{m-1+1} + t\sigma^{m-1+2} + \dots + t\sigma^{m-1+s} + t\sigma^{m-1}) \\
& \geq \Delta(M(y_{m+s+1}, y_{m+1}, t\sigma^{m-1+1} + t\sigma^{m-1+2} + \dots + t\sigma^{m-1+s}), \\
& \quad M(y_{m+1}, y_m, t\sigma^{m-1})) \\
& \geq \Delta(\Delta(M(y_{m+s+1}, y_{m+2}, t\sigma^{m-1+2} + \dots + t\sigma^{m-1+s}), \\
& \quad M(y_{m+2}, y_{m+1}, t\sigma^m), M(y_{m+1}, y_m, t\sigma^{m-1}))) \\
& \geq \Delta(\Delta(\Delta(M(y_{m+s+1}, y_{m+3}, t\sigma^{m-1+3} + \dots + t\sigma^{m-1+s}), \\
& \quad M(y_{m+3}, y_{m+2}, t\sigma^{m+1}), M(y_{m+2}, y_{m+1}, t\sigma^m), M(y_{m+1}, y_m, t\sigma^{m-1})))) \\
& \geq \dots \\
& \geq \overbrace{\Delta(\Delta(\dots(\Delta(M(y_{m+s+1}, y_{m+s}, t\sigma^{m-1+s}), M(y_{m+s}, y_{m+s-1}, t\sigma^{m+s-2}), \\
& \quad \dots, M(y_{m+1}, y_m, t\sigma^{m-1}))) \dots))}^{s\text{-times}} \\
& \geq \overbrace{\Delta(\Delta(\dots(\Delta(M(y_1, y_2, t\sigma^{m-1+s}/q^{m-1+s}), M(y_1, y_2, t\sigma^{m-2+s}/q^{m-2+s}), \\
& \quad \dots, M(y_1, y_2, t\sigma^{m-1}/q^{m-1}))) \dots))}^{s\text{-times}} \\
& \geq \overbrace{\Delta(\Delta(\dots(\Delta(M(y_1, y_2, t/\mu^{m-1+s}), M(y_1, y_2, t/\mu^{m-2+s}), \\
& \quad \dots, M(y_1, y_2, t/\mu^{m-1}))) \dots))}^{s\text{-times}} \\
& \geq \Delta_{i=m-1}^{m+s-1} M(y_1, y_2, t/\mu^i) \\
& \geq \Delta_{i=m-1}^{\infty} M(y_1, y_2, t/\mu^i).
\end{aligned}$$

It is obvious that

$$\lim_{n \rightarrow \infty} \Delta_{i=n}^{\infty} M(y_1, y_2, 1/\mu^i) = 1 \quad \text{implies} \quad \lim_{n \rightarrow \infty} \Delta_{i=n}^{\infty} M(y_1, y_2, t/\mu^i) = 1$$

for every $t > 0$. Now for every $t > 0$ and $\lambda \in (0, 1)$, there exists $m_1(t, \lambda)$ such that $M(y_{m+s+1}, y_m, t) > 1 - \lambda$ for every $m \geq m_1(t, \lambda)$ and $s \in \mathbb{N}$. Hence $\{y_n\}$ is a Cauchy sequence in X . Since X is complete, there exists a point z in X such that $\lim_{n \rightarrow \infty} y_n = z$ and this gives

$$\lim_{n \rightarrow \infty} Sx_{2n} = \lim_{n \rightarrow \infty} Tx_{2n-1} = \lim_{n \rightarrow \infty} Ax_{2n-2} = \lim_{n \rightarrow \infty} Bx_{2n-1} = z$$

for all $n \in \mathbb{N}$.

Now, suppose that pair (A, S) is compatible and S is continuous. Then by Lemma 2.1, A and S are reciprocally continuous. Hence $\lim_{n \rightarrow \infty} SAx_n = Sz$ and $\lim_{n \rightarrow \infty} ASx_n = Az$. Since pair (A, S) is compatible, we have

$$\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1$$

and hence $M(Az, Sz, t) = 1$. Thus $Az = Sz$.

Since $A(X) \subset T(X)$, there exists a point $v \in X$ such that $Az = Sz = Tv$. Take $\alpha = 1$ in (C2), we have

$$\begin{aligned} M(Az, Bv, qt) &\geq \min\{M(Tv, Bv, t), M(Sz, Az, t), M(Sz, Bv, t), \\ &\quad M(Tv, Az, t), M(Tv, Sz, t)\} \\ &= \min\{M(Az, Bv, t), M(Az, Az, t), M(Az, Bv, t), \\ &\quad M(Az, Az, t), M(Az, Az, t)\} \\ &\geq M(Az, Bv, t). \end{aligned}$$

Thus by Lemma 1.11, we have $Az = Bv$ and hence $Az = Sz = Tv = Bv$. Since A and S are R -weakly commuting mappings, there exists $R > 0$ such that $M(ASz, SAz, t) \geq (MAz, Sz, t/R) = 1$. Therefore, $ASz = SAz$ and $AAz = ASz = SAz = SSz$.

Similarly B and T are R -weakly commuting mappings, we have $BBv = BTv = TBv = TTv$.

Again take $\alpha = 1$, in (C2), we have

$$\begin{aligned} M(AAz, Bv, qt) &\geq \min\{M(Tv, Bv, t), M(SAz, AAz, t), M(SAz, Bv, t), \\ &\quad M(Tv, AAz, t), M(Tv, SAz, t)\} \\ &= \min\{M(Tv, Tv, t), M(AAz, AAz, t), M(AAz, Az, t), \\ &\quad M(Az, AAz, t), M(Az, AAz, t)\} \\ &\geq M(AAz, Az, t). \end{aligned}$$

By Lemma 1.11, we have $AAz = Az = SAz$. Hence Az is a common fixed point of A and S . Similarly we can prove $Bv = Az$ is a common fixed point of B and T . Hence, Az is a common fixed point of A, B, S and T .

Finally, in order to prove the uniqueness, suppose that $Av (\neq Az)$ is another fixed point of A, B, S and T . Then take $\alpha = 1$ in (C2), we have

$$\begin{aligned} M(Az, Av, qt) &= M(AAz, BAv, qt) \\ &\geq \min\{M(TAv, BAv, t), M(SAz, AAz, t), M(SAz, BAv, t), \\ &\quad M(TAv, AAz, t), M(TAv, SAz, t)\} \\ &= \min\{M(Av, Av, t), M(Az, Az, t), M(Az, Av, t), \\ &\quad M(Av, Az, t), M(Av, Az, t)\} \\ &\geq M(Az, Av, t). \end{aligned}$$

By Lemma 1.11, we have $Az = Av$. Therefore A, B, S and T have a unique common fixed point. This completes the proof. $\square$

If we put $S = T = I_X$ (I_X is the identity mapping) in Theorem 2.1, we get the following corollary.

Corollary 2.3. *Let (X, M, Δ) be a fuzzy metric space with continuous minimum t -norm of Hadžić-type. Let A and B be reciprocally continuous self-mappings on X satisfying*
there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq \min\{M(y, By, t), M(x, Ax, t), M(x, By, (2 - \alpha)t), \\ M(y, Ax, \alpha t), M(y, x, t)\}$$

for every $x, y \in X$ and $t > 0$, where $\alpha \in (0, 2)$.

Then A and B have a unique common fixed point in X .

Example 2.4. Let $X = [2, 20]$. Define for each $t > 0$, define

$$M(x, y, t) = \frac{t}{t + |x - y|}.$$

Then (X, M, Δ) is complete fuzzy metric space. Let A, B, S and T be self-mappings of X defined as

$$Ax = \begin{cases} 2 & \text{if } x = 2, \\ 3 & \text{if } x > 2, \end{cases} \quad Bx = \begin{cases} 2 & \text{if } x = 2 \text{ or } x > 5, \\ 6 & \text{if } 2 < x \leq 5, \end{cases}$$

$$Sx = \begin{cases} 2 & \text{if } x = 2, \\ 6 & \text{if } x > 2, \end{cases} \quad Tx = \begin{cases} 2 & \text{if } x = 2, \\ 12 & \text{if } 2 < x \leq 5, \\ x - 3 & \text{if } x > 5, \end{cases}$$

Then A, B, S and T satisfy all the conditions of Theorem 2.1 with $q \in (0, 1)$ and have a unique common fixed point $x = 2$.

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