Some Bounds on Nil Private Edge Domination

Number of a Graph

S. Robinson Chellathurai

Scott Christian College (Autonomous)
Nagercoil, Tamilnadu - 629 003, India

S. Kavitha

Arunachala College of Engineering for Women
Kanyakumari, Tamilnadu - 629 203, India

Copyright © 2014 S. Robinson Chellathurai and S. Kavitha. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

Let $G = (V, E)$ be a simple graph, set of edges $F \subseteq E$ in G is called an nil private edge dominating set if, F is an edge dominating set and every edge $e \in F$ has $pn(e, F) = \emptyset$. The minimum cardinality of a nil private edge dominating set of G is called nil private edge domination number and is denoted by $\gamma'_{nped}(G)$. In this paper some bounds on nil private edge domination number of a graph are obtained.

Keywords: nil private edge domination number, edge domination number

I. Introduction

Let $G = (V, E)$ be a connected undirected simple graph and let p and q denote order and size of the graph. The external private neighbor of the edge e with respect to the set F is defined by $pn(e, F) = N(e) - N(F - \{e\})$. A set $F \subseteq E$ is an edge dominating set if each edge in E is either an element of F or is adjacent to an edge in F . The edge domination number [4] $\gamma'(G)$ is the minimum

cardinality among all minimal edge dominating set of G . The set $F \subseteq E$ is said to be a nil private edge dominating set if it is an edge dominating set and for every $e \in F$, $pn(e, F) = \emptyset$. The minimum cardinality taken over all nil private edge dominating set of G is called nil private edge domination number and is denoted by $\gamma'_{nped}(G)$, we have the relation $\gamma'(G) \leq \gamma'_{nped}(G)$.

II. Nil private edge domination number of a graph

Definition 2.1 Let G be a non-trivial connected graph. A set $F \subseteq E$ is said to be a nil private edge dominating set if F is an edge dominating set and for every edge $e \in F$ has no external private edge neighbor in $E - F$. The minimum cardinality of a nil private edge dominating set is called the nil private edge domination number of a graph G and is denoted by $\gamma'_{nped}(G)$ and the corresponding set is called as γ'_{nped} - set of the graph G .

III. Some bounds on nil private edge domination number of a graph

In this section some bounds for the newly introduced parameter through the edge domination and other related parameter of the graph are obtained.

Theorem 3.1

For any connected graph G with

$$p > 3, \left\lceil \frac{q}{\Delta + 1} \right\rceil \leq \gamma'_{nped}(G) \leq 2q - p + 1.$$

Proof.

Since $\left\lceil \frac{q}{\Delta + 1} \right\rceil \leq \gamma'(G)$ [2] and $\gamma'(G) \leq \gamma'_{nped}(G)$, first inequality follows. For any connected graph $\gamma'_{nped}(G) \leq p - 1 = 2(p - 1) - p + 1 \leq 2q - p + 1$

Theorem 3.2

Let e be an edge of the graph G , with $\deg(e) = k$. If every edge in $N(e)$ is adjacent to at least one edge not in $N[e]$, then $\gamma'_{nped}(G) \leq q - k$.

Proof.

Since every edge in $N(e)$ is adjacent to at least one edge not in $N[e]$ then $E - N(e)$ is a nil private edge dominating set of G . Hence $\gamma'_{nped}(G) \leq |E - N(e)| = q - k$.

Theorem 3.3

Let F be a γ'_{nped} -set of a graph G , with $q > 3$. If $\gamma'_{nped}(G) = 2$, then $E - F$ is a $nped$ -set of G .

Proof.

Let F be a γ'_{nped} -set of G . Since $\gamma'_{nped}(G) = 2, q > 3$, $|E - F| \geq 2$, and each edge in $E - F$ is adjacent to both edges in F . Hence $E - F$ is a $nped$ -set of G .

Remark 3.4 The converse of the above Theorem 3.3 is not true.

Theorem 3.5

Let G be an acyclic graph with $q \geq 3$, then $\gamma'_{nped}(G) = 2$ if and only if G contains a complete bipartite graph $K_{1,n}$ $n \geq 2$.

Proof.

Let F be a γ'_{nped} -set of a graph G . Suppose $|F| = 2$, then every edge in $E - F$ is adjacent to both edges in F . Then all the edges are incident on a common vertex. Therefore G is a bipartite graph $K_{1,n}$. Conversely assume that G is a bipartite graph $K_{1,n}$. We want to prove that $\gamma'_{nped}(G) = 2$. In a graph G , each edge is incident on the same vertex. If we select one edge to F , then the edge contains a neighborhood private neighbor. So that $\gamma'_{nped}(G) > 1$, Hence $\gamma'_{nped}(G) \geq 2$. If we select any other edges to F from the given graph, it is $nped$ -set of a graph G so that $\gamma'_{nped}(G) \leq 2$. Hence $\gamma'_{nped}(G) = 2$.

Corollary 3.6

For any graph $K_{1,n}$ $n \geq 2$, $\gamma'_{nped}(G) = 2\gamma'(G)$.

Theorem 3.7

For a graph G , without isolated edges, $q \geq 2$ has a minimum edge dominating set, which is need not be γ'_{nped} -set of G .

Proof.

Every minimum edge dominating set F has an external private neighbor in $E - F$. Hence it is not said to be an $nped$ -set.

Theorem 3.8

Let G be a connected graph, a minimum edge dominating set is a γ'_{nped} -set if and only if $G = C_p$, $p = 2n$, $n = 1, 2, 3, \dots$

Proof. Given that G is an even cycle with $p = q$ edges. Let F be a γ' -set of a graph G , $|F| = \frac{q}{2}$. Also each edge in F has no private edge neighbor in $E - F$. Hence $|F| = \frac{q}{2} = \gamma'_{nped}(G)$.

Theorem 3.9

For a connected graph G , $\gamma'_{nped}(K_p) = \gamma'_{nped}(K_p - e)$, for $p \geq 3$.

Proof.

Let $e = uv$ be an edge in K_p . Also u and v are adjacent to every vertex in $K_p - e$. If we remove an edge e from K_p , then vertices u and v are adjacent with $(p-2)$ edges, all other vertices are adjacent with $p-1$ edges.

Theorem 3.10

Let G be a connected graph. If a γ'_{nped} -set of G is independent, then the following is true $\gamma'(G) + \gamma'_{nped}(G) \leq q$.

Proof.

Since F is edge independent, every edge in F is adjacent to at least one edge in $E - F$. Therefore $E - F$ is a dominating set.

Hence $\gamma'(G) \leq |E - F| = q - \gamma'_{nped}(G)$.

Theorem 3.11

Let F be a γ'_{nped} -set of G . Then F is a minimal dominating set if and only if F is an edge independent set of G .

Proof.

Suppose F is a minimal edge dominating set of G . Since F is a γ'_{nped} -set of G , for every $e \in F$ $pn[e, F] = \{e\}$. If F is not an independent set of G , then there exists at least two edges e_1 and e_2 in F , which are adjacent so that $pn[e, F] = \Phi$, which is a contradiction to the minimality of F . Converse part is obvious.

Theorem 3.12

For any tree T which is not a star or wounded spider, $\gamma'_{nped}(G) \geq \frac{q}{2}$.

Proof.

Let F be a γ'_{nped} -set of a tree T . Clearly an edge in $E - F$ is adjacent to at least two edges in F . Any two edges in F are adjacent to utmost one common edge in $E - F$, otherwise T contains a cycle so that $|F| \geq |E - F| = q - \gamma'_{nped}(G)$.

Hence $\gamma'_{nped}(G) \geq \frac{q}{2}$.

Theorem 3.13

For a connected graph G and does not contain pendent edges, then $\gamma'_{nped}(G) \leq \left\lceil \frac{2q}{3} \right\rceil$.

Proof.

Let F be a $\gamma'_{nped}(G)$ -set of a graph G . Every edge $e \in E - F$ is adjacent to two edges in F . If every edge in $E - F$ has exactly two private edge neighbors in F , then there will be utmost $\left\lceil \frac{2q}{3} \right\rceil$ edges in F . Suppose there exist an edge $e \in E - F$ has more than two private edge neighbors in F . The private edge neighbor of e form either 3 cycles with e or they are adjacent to some other edge in F as G contains pendent edges. In both cases choose two non adjacent private edge neighbors of e in F , say e' and e'' . Clearly $F \cup \{e\} - \{e', e''\}$ is an $nped$ -set which contradicts the minimality of F . Hence an edge e in $E - F$ cannot have more than two private edge neighbor in F .

Acknowledgements. The author wish to thank the referee for useful suggestions.

References

- [1] F. Harary, *Graph Theory*, Addison Wesley, Reading Mass (1969).
- [2] S. R. Jayaram, *Line domination in graphs*, Graph. Comb, 3(1987) 357-363.
<http://dx.doi.org/10.1007/bf01788558>
- [3] S. Robinson Chellathurai and S.Kavitha, *Private Edge Domination Number of a Graph*, J. Sci. Res 5(2), 2013. <http://dx.doi.org/10.3329/jsr.v5i2.12024>
- [4] Teresa W Haynes, Stephen T. Hedetniemi and Peter J Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc, New York (1998).

Received: December 9, 2014; Published: January 19, 2015