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Remark on the Sensitivity of Simulated Solutions of the Nonlinear Dynamical System to the Used Numerical Method

Robert Vrabel*, Vladimir Liska

Institute of Applied Informatics, Automation and Mechatronics
Faculty of Materials Science and Technology
Bottova 25, 917 01 Trnava, Slovakia
* Corresponding author

Juraj Vaclav

Technical University of Kosice, Slovakia

Abstract

In this short note we analyze the sensitivity of solutions to nonlinear second-order dynamical systems on the used numerical scheme. We show that numerical methods implemented in MATLAB can yield widely varying results for the same initial value problem for nonlinear dynamical systems.

Mathematics Subject Classification: 34C60

Keywords: dynamical system, oscillations, sensitivity, numerical simulation, MATLAB

1 Introduction

To study the nonlinear systems described by the system of differential equations two basic approaches have been developed - the analytical and numerical methods.

While the analytical methods are important to understand and predict phenomena in the behavior of the dynamical systems, numerical methods are techniques to approximate mathematical procedures. Approximations are needed because we either cannot solve the procedure analytically (integration of the nonlinear differential equations, for example) or because the analytical method is intractable, example is solving a system of a hundred simultaneous linear differential equations with constant coefficients.

In this note, by using the numerical schemes implemented in the MATLAB environment, we simulate the oscillations arising in the second-order differential equations of the form

$$\epsilon^2 y'' + f(t, y) = 0,$$

where ϵ is a small positive parameter and f is a continuous function.

Detailed analysis of this type equation may be found in the works [1, 2].

The objective of this note is to show the high sensitivity of solutions on the used numerical method in the sense, that by using the different numerical schemes to the same initial value nonlinear problem we can obtain qualitatively different, by the analytical theory admissible, solutions [1], see the Figs. 1-3 for illustration.

The object of interest is the dynamical system describing the singularly perturbed undamped oscillator with a continuous nonlinear restoring force

$$\begin{gathered} \epsilon^2 y'' + f(t, y) = 0, \\ y(-\delta) = y_0,\ y'(-\delta) = y_1 \end{gathered} \tag{1}$$

where

$$f(t, y) = \begin{cases} y^{4n+1} & \text{for } t \in [-\delta, 0] \\ y \prod_{i=1}^{2n} (y^2 - \mu^2 i^2 h^2(t)) & \text{for } t \in [0, \infty), \end{cases} \tag{2}$$

$[y_0, y_1]$ is an initial state, $y_\epsilon(\cdot, y_0, y_1)$ is a direct output, h is a positive continuous function on $[0, \infty]$, $n \in \mathbb{N}$, $\delta > 0$, and ϵ, $0 < \epsilon << 1$ is a singular perturbation parameter. It is instructive for the future to keep in mind the symmetric pitchfork-shaped manifold $f(t, y) = 0$. The parameter $\mu > 0$ is a constant determining the distance between pitchfork arms.

2 Numerical simulation

We use three solver functions implemented in the MATLAB, namely `ode45`, `ode23` and `ode113` to illustrate the problems with simulation of solutions to the nonlinear differential equation (1), (2) with $\delta = 0.02$, $n = 1$, $h(t) = t + \cos(t + \pi/2)$, $\mu = 0.5$, $\epsilon = 0.03$, and $y_0 = 0$, $y_1 = 0.095$, and $\epsilon = 0.03$.

- `ode45` - uses simultaneously fourth and fifth order Runge-Kutta (R-K) formulas to make error estimates and adjust the time step accordingly. MATLAB recommends that `ode45` is used as a first solver for nonstiff ODEs. Solver `ode45` is based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. It is a one-step solver in computing $y(t_n)$, it needs only the solution at the immediately preceding time point, $y(t_{n-1})$. In general, `ode45` is recommended as the best solver to apply as a first try for most problems.

- `ode23` - uses simultaneously second and third order R-K formulas to make estimates of the error, and calculate the time step size. Since the second and third order R-K require less steps, `ode23` is less expensive in terms of computation demands than `ode45`, but is also lower order. Solver `ode23` is an implementation of an explicit Runge-Kutta (2,3) pair of Bogacki and Shampine. It may be more efficient than `ode45` at crude tolerances and in the presence of moderate stiffness. Like `ode45`, `ode23` is a one-step solver.

- `ode113` - uses variable-order Adams-Bashforth-Moulton solver. Function `ode113` is recommended for problems with stringent error tolerances or for solving computationally intensive problems. It may be more efficient than `ode45` at stringent tolerances and when the ODE file function is particularly expensive to evaluate. Solver `ode113` is a multistep solver it normally needs the solutions at several preceding time points to compute the current solution [3, 4].

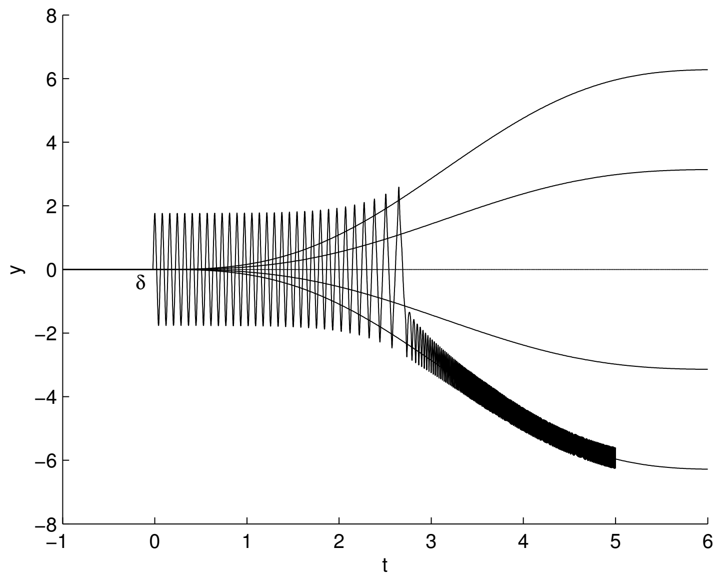

Figure 1: `ode45`: Numerical solution of (1), (2), where $\delta = 0.02$, $n = 1$, $h(t) = t + \cos(t + \pi/2)$, $\mu = 0.5$, $\epsilon = 0.03$, and $y(-0.02) = 0$, $y'(-0.02) = 0.095$, and $\epsilon = 0.03$.

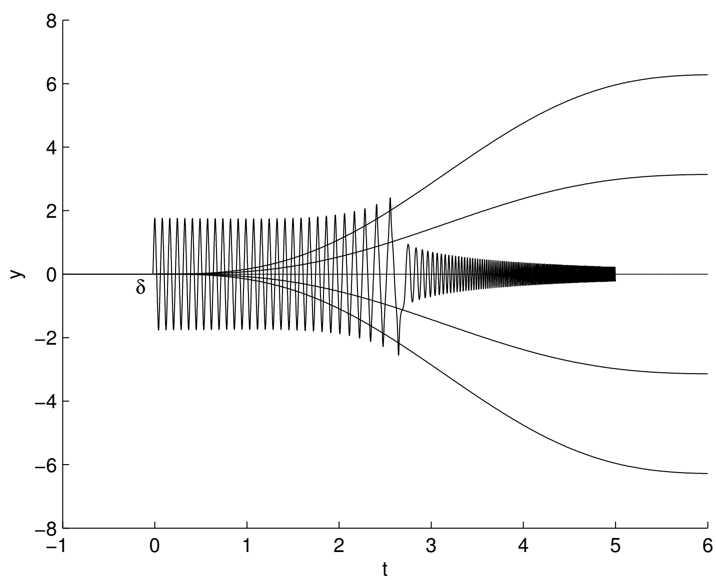

Figure 2: `ode23`: Numerical solution of (1), (2), where $\delta = 0.02$, $n = 1$, $h(t) = t + \cos(t + \pi/2)$, $\mu = 0.5$, $\epsilon = 0.03$, and $y(-0.02) = 0$, $y'(-0.02) = 0.095$, and $\epsilon = 0.03$.

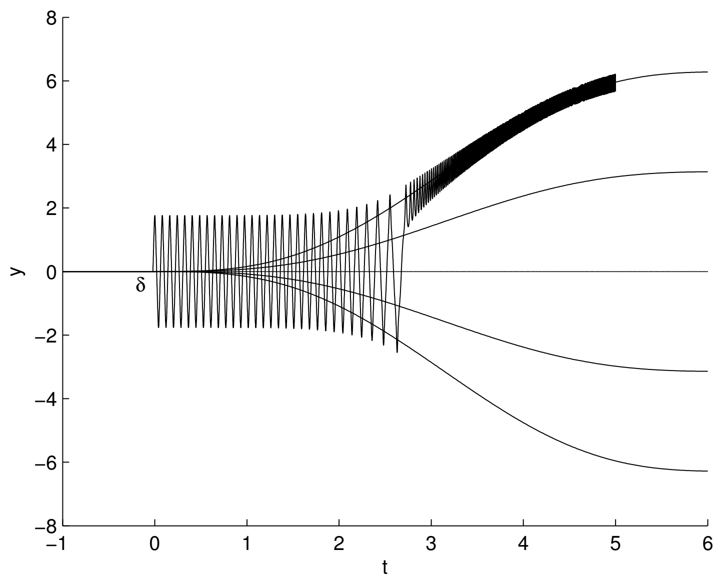

Figure 3: `ode113`: Numerical solution of (1), (2), where $\delta = 0.02$, $n = 1$, $h(t) = t + \cos(t + \pi/2)$, $\mu = 0.5$, $\epsilon = 0.03$, and $y(-0.02) = 0$, $y'(-0.02) = 0.095$, and $\epsilon = 0.03$.

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