Decentralized Stabilization of Interconnected Neutral Delay Large-Scale Systems¹

Jinjiang Yao

Department of Mathematics
Linyi Normal University
Linyi, Shandong, 276005, P. R. China
lysyyaojinjiang@yahoo.com.cn

Abstract

This paper is concerned with the decentralized stabilization problem for a class of interconnected neutral delay large-scale system. Based on the linear matrix inequality (LMI) design method, the problem we address is to design a decentralized state-feedback control law such that the interconnected neutral delay large-scale system can be stabilized. A simulation example is given to illustrate the effectiveness of the approach presented in this paper.

Mathematics Subject Classifications: TP 13

Keywords: Decentralized stabilization; Interconnected systems; Neutral delay

1 Introduction

The states changes of some systems are affected by not only delay state but also delay differential quotient, this kind of delay system is called to be neutral delay system. Many systems can be constructed by neutral differential equation, such as steam or water pipes, heater exchangers [1], lossless transmission lines and full wave equivalent circuit, equivalent electric path and so on. So the neutral delay systems are concerned widely in nearly years[1-6]. The research about neutral delay systems is concentrated in stability analysis and finding the conditions of asymptotically stable delay dependence or independence. Xu[4] solved the control problems of linear neutral delay system, and a design method of control based on the LMI method was provided. Based on the

¹This work was supported by Natural Science Foundation of China(Grant No. 10671086).

conversion methods of model and the LMI method, Han[5] gave a method to verify the stability of interconnected neutral delay large-scale system.

Because of the reliability and economical of the reality of decentralized control, it has been a very active point, and there were many researches about it[7-10]. It is very meaningful to do some decentralized control researches about interconnected neutral delay large-scale system which is made of finite subsystems; there were few reports about it. The decentralized control problems of interconnected neutral delay large-scale system are mainly talked about in this paper; we design a feedback control scaling law based on the method of LMI such that the interconnected neutral delay large-scale system can be stabilized. A simulation example is given to illustrate the effectiveness of the approach presented lastly.

2 The description of system

In this section, we consider a interconnected neutral delay large-scale system which is made of N subsystem. Its subsystem equation is

$$\dot{x}_i(t) = A_i x(t) + \sum_{j=1}^N A_{h_{ij}} x(t - h_{ij}) + A_{d_i} \dot{x}(t - d_i) + B_i u_i(t), \quad (1)$$

$$x_i(t) = \varphi_i(t), t \in [-l, 0], i = 1, 2, \dots, N,$$

where $x_i(t) \in R^{n_i}$ is state vector, $u_i(t) \in R^{m_i}$ is control impute, $A_i, A_{h_{ij}}, A_{d_i}, B_i$ are some matrixes which have right dimensions. $A_{h_{ij}}$ is a relevance matrix between the subsystems of j and i , scalars $h_{ij} > 0$ and $d_i > 0$ are delay constants, $l = \max\{h_{ij}, d_i\}$, $\varphi(t) \in R^n$ is original value vector function which is continuously differentiable.

We will design a memoryless state feedback control to every subsystem

$$u_i(t) = K_i x_i(t), \quad (2)$$

in order to make the closed-loop large-scale system

$$\dot{x}_i(t) = \bar{A}_i x(t) + \sum_{j=1}^N A_{h_{ij}} x(t - h_{ij}) + A_{d_i} \dot{x}(t - d_i), i = 1, 2, \dots, N, \quad (3)$$

asymptotically stable, $\bar{A}_i = A_i - B_i K_i$. The system (1) is called to be decentralized stabilization.

3 The design of decentralized controller

A condition of decentralized stabilization of system (1) is given by the following theorem.

Theorem 1. Considering interconnected neutral delay large-scale system (1), if there exists a matrix Y_i and positive definite matrix $P_i > 0, H_i > 0, Q_i > 0$ such that linear matrix inequality group

$$\begin{pmatrix} \sum_i T_{1i} & T_{2i} \\ T_{1i}^T & T_{2i} \end{pmatrix} \leq 0, \quad (4)$$

then the system (1) is decentralized stabilizable and $K_i = Y_i P_i^{-1}$, where

$$\begin{aligned} \sum_i &= P_i A_i^T + A_i P_i + Y_i^T B_i^T + B_i Y_i + Q_i + N H_i, \\ T_{1i} &= ((P_i A_i^T + Q_i + N H_i + Y_i^T B_i^T) A_{d_i}^T \quad P_i A_{h_{i1}}^T \cdots P_i A_{h_{in}}^T), \\ T_{2i} &= \text{diag}(Q_i - A_{d_i}(Q_i + N H_i) A_{d_i}^T, H_1 \cdots H_N). \end{aligned}$$

Proof Considering the condition of system of (1), let the control level to be (2), the closed-loop compound of interconnected neutral delay large-scale system (1) is (3), it is easy to know that the stability of the system (3) and the stability of

$$\dot{x}_i(t) = \bar{A}_i^T x(t) + \sum_{j=1}^N A_{h_{ij}}^T x(t - h_{ij}) + A_{d_i}^T \dot{x}(t - d_i) \quad (5)$$

are equal.

Assume $\wp_i(x_i(t)) = x_i(t) - A_{d_i}^T x(t - d_i)$, then system (5) is

$$\dot{\wp}_i(x_t) = \bar{A}_i^T x_i + \sum_{j=1}^N A_{h_{ij}}^T x(t - h_{ij}), i = 1, 2, \dots, N.$$

Because of the LMI (4), we know $Q_i - A_{d_i} Q_i A_{d_i}^T < 0$ easily. So

$$\wp_i(x_i(t)) = x_i(t) - A_{d_i}^T x(t - d_i)$$

is stable.

Considering the following Lyapunov-Krasovskii function

$$\begin{aligned} V(x_t, t) &= \sum_{i=1}^N [\wp^T(x_i(t)) P_i \wp(x_i(t)) \\ &\quad + \sum_{j=1}^N \int_{-h_{ij}}^0 x_j^T(t + \theta) H_i x_j(t + \theta) d\theta \\ &\quad + \int_{-d_i}^0 x_i^T(t + \theta) Q_i x_i(t + \theta) d\theta], \end{aligned}$$

we note $\wp_i(x_i(t)) = \wp_i, x_i(t - d_i) = x_{d_i}$ for short, then

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^N \{2\wp_i^T(x_i(t))P\dot{\wp}(x_i(t)) + \sum_{j=1}^N [x_j^T H_j x_j + x_j(t - h_{ij})^T H_j x_j(t - h_{ij})] \\
&\quad + x_i^T Q_i x_i - x_i(t - d_i)^T Q_i x_i(t - d_i)\} \\
&= \sum_{i=1}^N \{2\wp_i^T P_i \bar{A}_i^T (\wp_i + A_{d_i}^T x_{d_i}) + 2 \sum_{j=1}^N (\wp_i^T P_i A_{h_{ij}}^T x_j(t - h_{ij}) \\
&\quad + \sum_{j=1}^N [x_j^T H_j x_j - x_j(t - h_{ij})^T H_j x_j(t - h_{ij})] + x_i^T Q_i x_i - x_i(t - d_i)^T Q_i x_i(t - d_i)\} \\
&= \sum_{i=1}^N \{\wp_i^T (P_i \bar{A}_i^T + \bar{A}_i^T P_i) \wp_i + 2\wp_i^T P_i \bar{A}_i^T A_{d_i}^T x_{d_i} + 2 \sum_{j=1}^N \wp_i^T P_i A_{h_{ij}}^T x_j(t - h_{ij}) \\
&\quad + \sum_{j=1}^N [(\wp_j + A_{d_j}^T x_{d_j})^T H_j (\wp_j + A_{d_j}^T x_{d_j})] - \sum_{j=1}^N x_j^T (t - h_{ij}) H_j x_j(t - h_{ij}) \\
&\quad + [(\wp_i + A_{d_i}^T x_{d_i})^T Q_i (\wp_i + A_{d_i}^T x_{d_i})] - x_{d_i}^T Q_i x_{d_i}\} \\
&= \sum_{i=1}^N \{\wp_i^T (P_i \bar{A}_i^T + \bar{A}_i P_i + Q_i) \wp_i + \sum_{j=1}^N \wp_j^T H_j \wp_j + 2\wp_i^T (P_i \bar{A}_i^T + Q_i) A_{d_i}^T x_{d_i} \\
&\quad + 2 \sum_{j=1}^N \wp_j^T H_j A_{d_j}^T x_{d_j} + 2 \sum_{j=1}^N \wp_i^T P_i A_{h_{ij}}^T x_j(t - h_{ij}) - \sum_{j=1}^N x_j^T (t - h_{ij}) H_j x_j(t - h_{ij}) \\
&\quad + \sum_{j=1}^N x_{d_j}^T A_{d_j} H_j A_{d_j}^T x_{d_j} + x_{d_i}^T A_{d_i} Q_i A_{d_i}^T x_{d_i} - x_{d_i}^T Q_i x_{d_i}\} \\
&= \sum_{i=1}^N (\wp_i^T (P_i \bar{A}_i^T + \bar{A}_i P_i + Q_i) \wp_i + \sum_{i=1}^N \wp_i^T H_i \wp_i + 2\wp_i^T (P_i \bar{A}_i^T + Q_i) A_{d_i}^T x_{d_i} \\
&\quad + 2 \sum_{j=1}^N \wp_i^T H_i A_{d_j}^T x_{d_j} + 2 \sum_{j=1}^N \wp_i^T P_i A_{h_{ij}}^T x_j(t - h_{ij}) - \sum_{j=1}^N x_j^T (t - h_{ij}) H_j x_j(t - h_{ij}) \\
&\quad - x_{d_i}^T (Q_i - A_{d_i} (Q_i + N H_i) A_{d_i}^T) x_{d_i}).
\end{aligned}$$

Moreover, using $\alpha\beta + \beta^T\alpha^T \leq \alpha\alpha^T + \beta\beta^T$, α, β are matrix which have probable dimension, then

$$\dot{V} \leq \sum_{i=1}^N \wp_i^\top \Pi \wp_i,$$

where

$$\begin{aligned}
\Pi &= P_i \bar{A}_i^T + \bar{A}_i P_i + Q_i + N H_i \\
&\quad + (P_i \bar{A}_i^T + Q_i + N H_i) A_{d_i}^T (Q_i - A_{d_i} (Q_i + N H_i) A_{d_i}^T)^{-1} A_{d_i} \\
&\quad \times (P_i \bar{A}_i^T + Q_i + N H_i)^T + \sum_{j=1}^N P_i A_{h_{ij}}^T H_j^{-1} A_{h_{ij}} P_i.
\end{aligned}$$

If $\prod < 0$, then the proof could be finished by the stability theorem of [7, Th7.1], because difference operator is stable.

By the schur complementary lemma, it is easy to know that $\prod < 0$ is equal to

$$\begin{pmatrix} \Xi_i & \Gamma_{1i} \\ \Gamma_{1i}^T & \Gamma_{2i} \end{pmatrix} < 0,$$

where

$$\begin{aligned} \Xi_i &= P_i \bar{A}_i^T + \bar{A}_i^T P_i + Q_i + \sum_{j=1}^N H_i, \\ \Gamma_{1i} &= (P_i \bar{A}_i^T + Q_i + N H_i \quad P_i A_{h_{i1}}^T \cdots P_i A_{h_{iN}}^T) \frac{n!}{r!(n-r)!}, \\ \Gamma_{2i} &= \text{diag}(Q_i - A_{d_i}(Q_i + N H_i) A_{d_i}^T, H_1 \cdots H_N). \end{aligned}$$

Let $K_i P_i = Y_i$, then $\prod < 0$ from LMIs (4) and (7). $\square$

Remark. !!It is easy to transform the condition of (4) to LMI condition. In addition, the stability of difference operator can be seen in [4].

4 Example

Considering a interconnected neutral delay large-scale system which is made of two subsystem

$$\dot{x}_i(t) = A_i x(t) + \sum_{j=1}^2 A_{h_{ij}} x(t-h) + A_{d_i} \dot{x}(t-d) + B_i u(t), \quad i = 1, 2; j = 1, 2,$$

where

$$\begin{aligned} A_1 &= \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}, A_{h_{11}} = \begin{pmatrix} 0.5 & 0 \\ 0 & -0.2 \end{pmatrix}, A_{d_1} = \begin{pmatrix} -0.1 & 0.5 \\ 0.1 & 0 \end{pmatrix}, \\ B_1 &= 1, A_2 = \begin{pmatrix} -1 & 1.5 \\ 1 & -1 \end{pmatrix}, A_{h_{21}} = \begin{pmatrix} 0 & 0.2 \\ 0 & -0.1 \end{pmatrix}, A_{h_{22}} = \begin{pmatrix} 0.4 & 0 \\ 0.5 & 0 \end{pmatrix}, \\ A_{d_2} &= \begin{pmatrix} 0 & 0.5 \\ 0.1 & 0.2 \end{pmatrix}, B_2 = 1. \end{aligned}$$

It is to know that two subsystems are unstable, by solving the LMI (4), we have

$$\begin{aligned} P_1 &= \begin{pmatrix} 21.5916 & 0 \\ 0 & 31.1878 \end{pmatrix}, P_2 = \begin{pmatrix} 21.5916 & 0 \\ 0 & 31.1878 \end{pmatrix}, H_1 = \begin{pmatrix} 38.2569 & 0.6315 \\ 0.6315 & 27.3701 \end{pmatrix}, \\ H_2 &= \begin{pmatrix} 39.2966 & 0 \\ 0 & 39.2966 \end{pmatrix}, Q_1 = \begin{pmatrix} 47.4751 & 0.7761 \\ 0.7761 & 34.0535 \end{pmatrix}, Q_2 = \begin{pmatrix} 47.4175 & 2.9265 \\ 2.9265 & 34.3461 \end{pmatrix}, \\ Y_1 &= \begin{pmatrix} -60.3728 & 0 \\ 0 & 31.1878 \end{pmatrix}, Y_2 = \begin{pmatrix} -81.8122 & -5.4272 \\ 0.7544 & -68.3873 \end{pmatrix}. \end{aligned}$$

Figure 1.

Moreover

$$K_1 = Y_1 P_1^{-1} = \begin{pmatrix} -2.7961 & 0.6312 \\ -1.5755 & -1.1423 \end{pmatrix}, K_2 = Y_2 P_2^{-1} = \begin{pmatrix} -3.7891 & -0.1740 \\ 0.0349 & -2.1607 \end{pmatrix},$$

the decentralized state-feedback control laws are $u_1 = k_1 x_1$ and $u_2 = k_2 x_2$, choose initial condition as $x_1(t_0) = [3, 2]^T$, $x_2(t_0) = [4, 5]^T$, $h_{ij} = 0.5$, $d_i = 0.2$.

The state of two subsystems are presented in figure 1, and the real line is the state of subsystem one, the imagine line is the state of subsystem of another. It is obviously that the system could be stabilized by the designed decentralized state-feedback control law.

5 Conclusion

Decentralized stabilization problem for a class of interconnected neutral delay large-scale system is considered in this paper, a condition of asymptotically stable of interconnected neutral delay large-scale system is given and the result is given in the form of LMI. It is easy to solve and could be applied easily. But we only considered a simple interconnected neutral delay large-scale system, the complicated interconnected neutral delay large-scale system will be studied in the nearly future.

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Received: January 9, 2007