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Robust Exponential Stability of Mild Solutions to Impulsive Stochastic Neutral Integro-Differential Equations with Delay

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Abstract

In this paper, we study the robust properties of exponential stability of mild solutions for some classes of impulsive stochastic integro-differential perturbed system with function of finite delay time and supported this stability by some important result.

Keywords: stability of mild solutions, impulsive stochastic integro- differential equations

1. Introduction

The Impulsive systems arise naturally in various fields, such as mechanical systems and biological systems, economics, etc. see [8]. Impulsive dynamical systems exhibit the continuous evolutions of the states typically described by ordinary differential equations coupled with instantaneous state jumps or impulses. And the presence of impulses implies that the trajectories of the system do not necessarily preserve the basic properties of the non-impulsive dynamical systems. To this end the theory of impulsive differential systems has emerged as an important area of investigation in applied sciences and impulsive stochastic integro-differential equations studied in [1].In the last few years many papers have been published about the stability of impulsive integro-differential systems [3].In particular, the exponential stability of mild solutions of various stochastic delay differential equations has been established [7].The conditions ensuring the exponential stability given by [6] and[5]. The aim of this paper is to study the existence and exponential stability of a class of impulsive control stochastic integro-differential equations of mild solutions by using a new integral inequality.

2. Preliminaries

Let $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions. Moreover, let X,Y be two real separable Hilbert space and let L(Y.X) denote the space of all bounded linear operators from Y to X and let L(Y.X) denoted the space of all bounded operators from Y to X . For simplicity, we use the notation |.| to denote the norm in X,Y and ||·|| to denote the operator norm in L(X.X) and L(Y.X).Let $\langle . \rangle_x, \langle . \rangle_Y$ denote the inner products of X,Y, respectively . Let {ŵ (t): t≥0} denote a Y-valued Wiener process defined on the probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with covariance operator Q ,that is , $E\langle w(t), x\rangle_Y\, E\langle w(s), y\rangle_X = (t \wedge s)\langle Qx, y\rangle_Y$, for all x, y $\in$ Y, where Q is positive , self –adjoin a, trace class operator on Y. In particular, we dente by w(t) Y-valued Q- Wiener process with respect to $\{F_t\}_{t\geq 0}$ .We assume that there exists a complete orthonormal system $\{e_i\}$ in Y, a bounded sequence of nonnegative real numbers λ_i such that $Qe_i = \lambda_i e_i \; i = 1,2,...,$ and a sequence $\{\beta_i\}_{i\geq 1}$ of independent Brownian motions such that $\langle w(t), e\rangle = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\, \langle e_i, e\rangle \beta_i(t), e \in Y$, and $F_i = F_i^W$,where F_i^W is the ϭ-algebra generated by{w(t):0≤s≤t}.Let $L_2^0 = L_2(Q^{1\backslash 2}Y; X)$ be the space of all Hilbert–Schmidt operators from $Q^{1\backslash 2}Y$ *to* X with the inner product $\langle z, \xi\rangle_{L_2^0} = tr[zQ\xi]$[4]. Suppose that T(t) is an analytic semigroup with its infinitesimal generator A; for literature relating to semigroup theory, we suggest Pazy[2] .We suppose 0 $\in$ p (A),the resolvent set of –A. For any α$\in$[0,1], it is possible to define the fractional power $(-A)^{\alpha}$ which is a closed linear operator with its domain $D(((-A)^{\alpha}))$ .In this paper we consider the following impulsive stochastic neutral integro-differential equations with finite delay:

$$\begin{cases} d[x(t)-z(t,x(t-p(t)))]=Ax(t)+Bu(t)+f(t,x(t-\tau(t))),\int_0^t f_1(t,s,x(s-\tau(s)))ds,\int_0^t f_2(t,s,x(s-\tau(s)))dw(s)) \\ \qquad +g((t,x(t-\delta(t))),\int_0^t g_1(t,s,x(s-\delta(s)))ds,\int_0^t g_2(t,s,x(s-\delta(s)))dw(s))dw(t)/dt \\ \qquad +\sigma(x(t-\hat{\sigma}(t))dw(t)/dt, \\ \Delta x(t_k)=x(t_k^+)-x(t_k^-)=I(x(t_k^-)), \\ x_0(.)=\varphi\in C^b_{F_0}([-\tau,0],X), \end{cases} \tag{1}$$

In a real separable Hilbert space X, where u : $R_+ \times C([-\iota, 0],X) \to X$, f : $R_+ \times C([-\iota, 0],X) \to X$, g: $R_+\times C([-\iota, 0],X) \to L(X, Y)$, f_1, g_1: $R_+ \times C([-\iota, 0],X) \to X$, f_2, g_2: $R_+\times C([-\iota, 0],X) \to L(X, Y)$, $\sigma: R_+\times C([-\tau, 0],X)\to L(X, Y)$ are all Borel measurable; p: $R_+\to [-\iota, 0]$, ι: $R_+\to [\iota, 0]$, δ : $R_+\to [\iota, 0]$, $\hat{\sigma}: R_+ \to (0, \tau)$ are continuous ; A is the infinitesimal generator of semigroup of bounded linear operators T(t), $t \geq 0$ in X; B : U→X is a linear bounded operator $u(\cdot) \in U$ Hilbert space of admissible control functions and; I_k: X→X,. Furthermore, the fixed moments of time t_k satisfy $0< t_1 < ... < t_m < \lim_{k\to\infty}$, $t_k =\infty$, x (t^+_k) and $x(t^-_k)$ represent the right and left limits of x(t) at $t = t_k$, respectively. Also, $\Delta x (t_k) = x (t^+_k) - x(t^-_k)$, represents the jump in the state x at time t_k with I_k determining the size of the jump. Let $\tau > 0$ and C = C([–τ, 0];X) denote the family of all right continuous functions with left-hand limits η from [–τ, 0] to X . The space C is assumed to be equipped with the norm$\| \eta \|_C = \sup_{\theta\in[-\tau\, 0]} |\eta(\theta)|$. Here C^b_{F0} ([–τ, 0], X) is the family of all almost surely bounded, F_0-measurable, continuous random variables from [–τ, 0] to X.

Theorem 2.1[2]: Let T (t) be a C_0- semigroup generated by A. Then the following hold:
i. For each $x_0\in D(A)$; T(t) $x_0\in D(A)$ (domain of A) and AT(t) x_0 = T(t) A x_0,$\forall$ t≥ 0; ii. For each $x_0 \in D(A)$ and T(t) $x_0\in D(A)$; (d/dt)(T(t)x_0) = AT(t) x_0 = T(t) Ax_0

Theorem 2.2[2]: Let X be a Banach space and let A be the infinitesimal generator of C_0 semigroup T(t) on X, satisfying $\| T(t) \| \leq Me^{wt}$.
If B is a bounded linear operator on X then A+B is the infinitesimal generator of C_0 semigroup S (t) satisfying $\| S(t) \| \leq Me^{(w+M\|B\|)t}$

Definition 2.1: Aprocess {x (t), t ∈ [0, T]}, 0 ≤ T <∞, is called a mild solution of Eq. (1) if (i) x(t) is adapted to F_t , t ≥ 0 with $\int_0^T |x(t)|^P dt \prec \infty \ a.s;$
(ii) x (t) ∈ X has cadlag paths on t ∈ [0,T] a.s. and for each t ∈ [0,T], x(t) satisfies the integral equation

$$x(t)=T(t)[\varphi(0)-z(0,x(-p(0))+z(t,x(t-p(t))]+\int_0^t AT(t-s)z(t,x(t-p(t))ds$$
$$+\int_0^t T(t-s)Bu(s)ds+\int_0^t T(t-s)f(t,x(s-\tau(s))),\int_0^t f_1(t,s,x(s-\tau(s)))ds,\int_0^t f_2(s,x(s-\tau(s)))dw(s))d(s)$$
$$+\int_0^t T(t-s)g((t,x(s-\delta(s),\int_0^t g_1(t,s,x(s-\delta(s)))ds,\int_0^t g_2(t,s,x(s-\delta(s)))dw(s))dw(s)$$
$$+\int_0^t T(t-s)\sigma(x(s-\hat{\sigma}(s))dw(s)+\sum_{0\prec t_k\prec t} T(t-t_k)I_K(x(t_k^-)) \qquad (2)$$

And $x_0(.)=\varphi \in C^b_{F0}([-\tau, 0], X)$.

Definition 2.2: Let $p \geq 2$ be an integer. Equation (1) is said to be exponentially stable in the pth mean if for any initial value ϕ, there exists a pair of positive constants λ and K_0 such that
$E|x(t)|^p \leq k_0 \|\varphi\|^p_C e^{-\lambda t}$ *for* $t \geq 0$ In particular, if p = 2, then Eq. (1) is said to be mean-square exponentially stable.
To establish the exponential stability of the mild solution of Eq. (1), we employ the following assumptions:

(H_1) A is the infinitesimal generator of a semigroup of bounded linear operators Š (t), $t \geq 0$, in X satisfying $|$Š$(t)| \leq Me^{-at}$, $t \geq 0$, for some constants $M \geq 1$ and $0 < a \in R+$.
(H_2) $u:[0,\infty] \to U$ is a feedback control input function of U Hilbert space of admissible control Such that $u(t)= -\hat{w}x(t)$ where $\hat{w}$ is bounded linear operator .
(H_3) $(A-B\hat{w})$ the perturbation infinitesimal generator of a semigroup of bounded linear operators T(t), $t \geq 0$, in X satisfying $|T(t)| \leq M_1 e^{(w+\|B\hat{w}\|)t}$, $t \geq 0$, for some constants $M_1 \geq 1$ and $0 < (w-\|B\hat{w}\|) \in R^+$.
(H_4) The mappings f and g satisfy the following Lipschitz condition: there exists a constant K for any x, y ∈ X and $t \geq 0$ such that

$$\tilde{F}(t,x)=f(t,x(t),\int_0^t f_1(t,s,x(s)ds,\int_0^t f_2(t,s,x(s))dw(s)),$$
$$\tilde{G}(t,y)=g((t,x(t),\int_0^t g_1(t,s,x(s)ds,\int_0^t g_2(t,s,x(s))dw(s)),$$
$$\left|\tilde{F}(t,x)-\tilde{F}(t,y)\right| \leq k|x-y|, \quad \left\|\tilde{G}(t,x)-\tilde{G}(t,y)\right\| \leq k|x-y|, \quad \left\|\sigma(t,x)-\sigma(t,y)\right\| \leq k|x-y|.$$

(H_5) The mapping z(t, x) satisfies that there exists a number $\alpha \in [0, 1]$ and a positive constant K such that for any x, y ∈ X and $t \geq 0$, $z(t, x) \in D((-(A-B\hat{w})^\alpha)$ and $\left|(-(A-B\hat{w}))^\alpha z(t,x) - (-(A-B\hat{w}))^\alpha z(t,y)\right| \leq \bar{K}|x-y|$

(H_6) There exists a constant q_k such that $|I_k(x) - I_k(y)| \leq q_k\, |x - y|$, k = 1, . . . ,m, for each x, y∈ X. Moreover, for the purposes of stability, we always assume that z (t, 0) = 0, $\tilde{F}(t)=0, \tilde{G}(t)=0, I_k(0)=0\,(k=1,2,...,m)$ Hence Eq.(1) has a trivial solution when φ =0.The solution of (1) by hypothesis is

$$x(t) = T(t)[\varphi(0) - z(0, x(-p(0)) + z(t, x(t-p(t))] + \int_0^t (A - B\hat{w})T(t-s)z(t, x(t-p(t))ds$$

$$+ \int_0^t T(t-s)\tilde{F}(t,x)ds + \int_0^t T(t-s)\tilde{G}(t,y)dw(s) + \int_0^t T(t-s)\sigma(x(s-\hat{\sigma}(s))dw(s)$$

$$+ \sum_{0 \prec t_k \prec t} T(t - t_k) I_K(x(t_k^-)) \quad (3)$$

Lemma2.1: If (H_3) holds, then for any β ∈ (0, 1]:
(i) For each x ∈ D(-(A-Bŵ))β), T(t)(−(A-Bŵ))βx = (−(A-Bŵ))βT(t)x;
(ii) There exist positive constants $M_\beta > 0$ and $(w - \|B\hat{w}\|) \in R^+$ such that $\left\|(-(A-B\hat{w}))^{\beta} T(t)\right\| \leq M_\beta t^{-\beta} e^{-(w - \|B\hat{w}\|)t}$, t > 0.

3 Stability of mild solutions [5] [6] [7]

In this section, to establish sufficient conditions ensuring the exponential stability in p moment (p ≥ 2) for a mild solution to Eq. (1), we firstly establish a new integral inequality to overcome the difficulty when the neutral term and impulsive effects are present.

Lemma 3.1[5] For any γ > 0, assume that there exist some positive constants α_i (i = 1, .., m), β_k (k = 1, 2, . . . ,m) and a function ψ : [−τ ,∞)→[0,∞) such that $\psi(t) \leq \alpha e^{-\gamma t}$ for t ∈ [−τ, 0]

$$\psi(t) \leq \alpha_1 e^{-\gamma t} \quad (4)$$

and

$$\psi(t) \leq \alpha_1 e^{-\gamma t} + \alpha_2 \sup_{\theta \in [-\tau,0]} \psi(t+\theta) + \alpha_3 \int_0^t e^{-\gamma(t-s)} \sup_{\theta \in [-\tau,0]} \psi(t+\theta) + \sum_{t_k \prec t} \beta_k e^{-\gamma(t-t_k)} \psi(t_k^-) \quad (5)$$

for each $t \geq 0$, *If*

$$\alpha_2 + \frac{\alpha_1}{\gamma} + \sum_{k=1}^{m} \beta_k \prec 1, \quad (6)$$

then

$$\psi(t) \leq M_0 e^{-\gamma t} \text{ for } t \geq -\tau \quad (7)$$

where $\lambda > 0$ is the unique solution to the equation: $\alpha_2 e^{\lambda\tau} + \alpha_3 e^{\lambda\tau}/(\gamma - \lambda) + \sum_{k=1}^{m}\beta_k = 1$ and

$$M_0 = Max\left\{\alpha_1, \frac{\alpha_1(\gamma-\lambda)}{\alpha_3 e^{\lambda\tau}}\right\} \succ 0$$

Theorem 3.1 If (H_3)-(H_6) hold for some $\alpha \in (1/p, 1]$, $p \geq 2$, then the mild solution of Eq. (1) is exponentially stable, provided

$$ak(1-k)^{p-1} + 8^{p-1}M_{1-\alpha}^{p}\bar{K}^{p}(\text{w-}\|\text{B}\hat{\text{w}}\|)^{1-p\alpha}(\Gamma(1+q\alpha-q))^{\frac{p}{q}}$$
$$+4^{p-1}M^{p}K^{p}(\text{w-}\|\text{B}\hat{\text{w}}\|)^{1-p} + 4^{p-1}c_p M^{p}K^{p}\left(\frac{2(\text{w-}\|\text{B}\hat{\text{w}}\|)(p-1)}{p-2}\right)^{1-\frac{p}{2}}$$
$$+4^{p-1}c_p M^{p}K^{p}\left(\frac{(p-2)}{2(p-1)(\text{w-}\|\text{B}\hat{\text{w}}\|)}\right)^{(\frac{p-2}{2})}$$
$$+4^{p-1}(\text{w+}\|\text{B}\hat{\text{w}}\|)M^{p}(1-K)^{p-1}\left(\sum_{k=1}^{m}q_k\right)^{p} \prec (\text{w-}\|\text{B}\hat{\text{w}}\|)(1-k)^{p-1} \qquad (8)$$

where $c_p = (p(p-1)/2)^{p/21}$, $\kappa = \bar{K}\,|(-(\text{w+}\|\text{B}\hat{\text{w}}\|))^{-\alpha}|$ and $M_{1-\alpha}$ is defined in Lemma3.1.

Proof From the condition (8), we can always find a number $\varepsilon > 0$ small enough such that

$$(\text{w-}\|\text{B}\hat{\text{w}}\|)k(1-k)^{p-1} + 8^{p-1}M_{1-\alpha}^{p}\bar{K}^{p}(\text{w-}\|\text{B}\hat{\text{w}}\|)^{1-p\alpha}(\Gamma(1+q\alpha-q))^{\frac{p}{q}}$$
$$+4^{p-1}M^{p}K^{p}(\text{w-}\|\text{B}\hat{\text{w}}\|)^{1-p} + 4^{p-1}c_p M^{p}K^{p}\left(\frac{2(\text{w-}\|\text{B}\hat{\text{w}}\|)(p-1)}{p-2}\right)^{1-\frac{p}{2}}$$
$$+4^{p-1}c_p M^{p}K^{p}\left(\frac{(p-2)}{2(p-1)(\text{w-}\|\text{B}\hat{\text{w}}\|)}\right)^{(\frac{p-2}{2})} + 4^{p-1}(\text{w-}\|\text{B}\hat{\text{w}}\|)M^{p}(1-K)^{p-1}\left(\sum_{k=1}^{m}q_k\right)^{p}$$
$$\prec (\text{w-}\|\text{B}\hat{\text{w}}\|)(1-k)^{p-1}$$

On the other hand, recall the inequalities $|u-v|^p \leq |u|^p/\varepsilon^{p-1} + |v|^p/(1-\varepsilon)^{p-1}$ and $|u+v|^p \leq |u|^p(1+\varepsilon)^{p-1} + |v|^p(1+1/\varepsilon)^{p-1}$ for u, v $\in$ X, $\in \succ 0$ .Then, for any $x_1, x_2, \ldots, x_7$.

$$E|x(t)|^p \leq \frac{1}{k^{p-1}}E|s(t,x(t-p(t)))|^p + \frac{1}{(1-k)^{p-1}}E|x(t)-s(t,x(t-p(t)))|^p$$

$$|x_1+x_2+x_3+x_4+x_5+x_6+x_7|^p \leq 4^{p-1}(1+1/\varepsilon)^{p-1}|x_1|^p + 8^{p-1}(1+\varepsilon)^{p-1}(|x_2|^p + |x_3|^p)$$
$$+4^{p-1}|x_4|^p + 4^{p-1}|x_5|^p + 4^{p-1}|x_6|^p + 4^{p-1}|x_7|^p \qquad (9)$$

From (3) and (9)

$$E|x(t)|^p \leq 1/k^{p-1}E|z(t,x(t-p(t)))|^p + 1/(1-k)^{p-1}E|x(t)-z(t,x(t-p(t)))|^p$$

$$E\left|x(t)\right|^{p} \le \frac{1}{k^{p-1}} E\left|z(t,x(t-p(t)))\right|^{p} + \frac{1}{(1-k)^{p-1}} E \left| \begin{array}{l} T(t)[\varphi(t)-z(0,x(t-p(0)))+\int_0^t (A+B\hat{w})T(t-s)z(s,x(s-p(s)))ds+\int_0^t T(t-s)\tilde{F}(s,x)ds \\ +\int_0^t T(t-s)\tilde{G}(s,y)dw(s)+\int_0^t T(t-s)\sigma(x(s-\hat{\sigma}(s))dw(s)+\sum_{0\prec t_k\prec t} T(t-t_k)I_K(x(t_k^-)) \end{array} \right|^{p}$$

$$\le \frac{1}{k^{p-1}} E\left|z(t,x(t-p(t)))\right|^{p} + \frac{1}{(1-k)^{p-1}} \{4^{p-1}(1+1/\varepsilon)^{p-1} E\left|T(t)\varphi(0)\right|^{p}$$

$$+8^{p-1}(1+\varepsilon)^{p-1} E\left|T(t)z(0,x(t-p(0)))\right|^{p} + 8^{p-1}(1+\varepsilon)^{p-1} E\left|\int_0^t (A+B\hat{w})T(t-s)z(s,x(s-p(s)))ds\right|^{p}$$

$$+4^{p-1} E\left|\int_0^t T(t-s)\tilde{F}(s,x)ds\right|^{p} + 4^{p-1} E\left|\int_0^t T(t-s)\tilde{G}(s,y)dw(s)\right|^{p}$$

$$+4^{p-1} E\left|\int_0^t T(t-s)\sigma(x(s-\hat{\sigma}(s))dw(s)\right|^{p} + 4^{p-1} E\left|\sum_{0\prec t_k\prec t} T(t-t_k)I_K(x(t_k^-))\right|^{p} \}$$

$$= \frac{1}{k^{p-1}} F_0 + \frac{1}{(1-K)^{p-1}} \sum_{i=1}^{7} F_i \qquad (10)$$

Now we compute the right –hand terms of (10) .Firstly, by (H_3) and (H_5), we can easily obtain

$$F_0 \le K^{p} \sup_{\theta\in[-\tau,0]} E\left|x(t+\theta)\right|^{p}, \qquad (11)$$

$$F_1 \le 4^{p-1}(1+1/\varepsilon)^{p-1} M^{p} e^{-p(A-B\hat{w})t} E\left\|\varphi\right\|_C^{p}, \qquad (12)$$

$$F_2 \le 8^{p-1}(1+\varepsilon)^{p-1} M^{p} \left|(-(A-B\hat{w}))^{\alpha}\right|^{p} E\left\|\varphi\right\|_C^{p}, \qquad (13)$$

By (H_6) and the Holder inequality, for p≥2, 1<q≤2, $\frac{1}{p}+\frac{1}{q}=1$, we have

$$F_7 \le 4^{p-1} E\left|\sum_{0\prec t_k\prec t} T(t-t_k)I_K(x(t_k^-))\right|^{p} \le 4^{p-1} E\Big(\sum_{0\prec t_k\prec t} Me^{-(\text{w-}\|B\hat{w}\|)(t-t_k)} q_K \left|x(t_k^-)\right|\Big)^{p}$$

$$\le 4^{p-1} M^{p} E\Big(\sum_{0\prec t_k\prec t} q_k^{1/p} q_k^{1/q} e^{-(\text{w-}\|B\hat{w}\|)(t-t_k)} \left|x(t_k^-)\right|\Big)^{p}$$

$$\le 4^{p-1} M^{p} \Big(\sum_{0\prec t_k\prec t} q_k\Big)^{p/q} \sum_{0\prec t_k\prec t} q_k e^{-p(\text{w-}\|B\hat{w}\|)t} E\left|x(t_k^-)\right|^{p}. \qquad (14)$$

By (H_5), lemma3.1 and Holder inequality,

$$F_3 \le 8^{p-1}(1+\varepsilon)^{p-1}E\left(\int_0^t \left|(-(A-B\hat{w}))^{\alpha}T(t-s)(-(A-B\hat{w}))^{\alpha}z(s,x(s-p(s)))ds\right|\right)^p$$

$$\le 8^{p-1}(1+\varepsilon)^{p-1}M_{\alpha-1}^{p}\bar{K}^{p}\left(\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}(t-s)^{q\alpha-q}ds\right)^{p/q}$$

$$\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}E\left|x(s-p(s))\right|^p ds$$

$$\le 8^{p-1}(1+\varepsilon)^{p-1}M_{1(\alpha-1)}^{p}\bar{K}^{p}(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)^{1-p(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)}(\Gamma(1+q\alpha-q))^{p/q}$$

$$\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E|x(s-\theta)|^p ds. \tag{15}$$

Similar to (15), by (H_4) And Holder inequality,

$$F_4 \le 4^{p-1}M_1^{\,p}K^{p}(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)^{1-p}\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds \tag{16}$$

By Da prato and zabczyk, similar to (15), by (H_4) and Holder inequality,

$$F_5 \le 4^{p-1}C_pM_1^{\,p}K^{p}\left(\frac{2(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(p-1)}{p-2}\right)^{1-p/2}\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds \tag{17}$$

By Da prato and zabczyk[4], similar to (15), by (H_4) and Holder inequality,

$$F_6 \le 4^{p-1}C_pM_1^{\,p}K^{p}\left(\frac{(p-2)}{2(p-2)(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)}\right)^{(p-2)/2}$$

$$\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds \tag{18}$$

$$E\left|x(t))\right|^p \le K^p\sup_{\theta\in[-\tau,0]}E\left|x(t+\theta)\right|^p+1/(1-k)^{p-1}\{4^{p-1}(1+1/\varepsilon)^{p-1}M^{p}e^{-p(A-B\hat{w})t}E\left\|\varphi\right\|_C^p$$

$$+8^{p-1}(1+\varepsilon)^{p-1}M^{p}\left|(-(A-B\hat{w}))^{\alpha}\right|^p E\left\|\varphi\right\|_C^p+8^{p-1}(1+\varepsilon)^{p-1}M_{1(\alpha-1)}^{p}\bar{K}^{p}(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)^{1-p(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)}(\Gamma(1+q\alpha-q))^{p/q}$$

$$\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds+4^{p-1}M_1^{\,p}K^{p}(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)^{1-p}$$

$$\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds+4^{p-1}C_pM_1^{\,p}K^{p}\left(\frac{2(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(p-1)}{p-2}\right)^{1-p/2}$$

$$\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds+4^{p-1}C_pM_1^{\,p}K^{p}\left(\frac{(p-2)}{2(p-2)(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)}\right)^{(p-2)/2}$$

$$\times\int_0^t e^{-(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)(t-s)}\sup_{\theta\in[-\tau,0]}E\left|x(s-\theta)\right|^p ds+4^{p-1}M^{p}\Big(\sum_{0\prec t_k\prec t}q_k\Big)^{p/q}\sum_{0\prec t_k\prec t}q_ke^{-p(\mathrm{w}-\|\mathrm{B}\hat{\mathrm{w}}\|)t}E\left|x(t_k^-)\right|^p\} \tag{19}$$

This, together with lemma 3.1 and (8) gives that there exist two positive constants M_0 and $\lambda \in (0, (w - \| B\hat{w} \|))$ such that $E|x(t)|^p \le Me^{-\lambda t}$ for any $t \ge -\tau$ .This completes the proof.

Corollary3.1: If (H_3)-(H_6) hold for some $\alpha \in (1/2,1)$, then the mild solution of Eq.(1) is mean-square exponentially stable, provided

$$(w- \| B\hat{w} \|)\bar{k}(1-\bar{k})\left|(-(w- \| B\hat{w} \|))^{-\alpha}\right| + 8M_{1-\alpha}^{2}\bar{K}^{2}(w- \| B\hat{w} \|)^{1-2\alpha}\Gamma(2\alpha-1) + 4M^{2}K^{2}(w- \| B\hat{w} \|)^{-1}$$
$$+4M^{2}K^{2}+4(w+ \| B\hat{w} \|)M^{2}(1-K)(\sum_{k=1}^{m} q_k)^2 \prec (w+ \| B\hat{w} \|)((1-\bar{k}(\left|-(w+ \| B\hat{w} \|))^{-\alpha}\right|.$$

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