Solution of Generalized Fractional Kinetic Equation in Terms of Special Functions

Ahmad Faraj, Tariq Salim, Safaa Sadek and Jamal Ismail

Al-Azhar University-Gaza, Palestine

Copyright © 2014 Ahmad Faraj, Tariq Salim, Safaa Sadek and Jamal Ismail. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

This paper is devoted to introduce an analytical method for solving generalized fractional kinetic equation in terms of the generalized K_4 - function, the generalized M – Series and the generalized Mittag – Leffler type function, this method is depending on Laplace transform operator technique.

Keywords: Generalized fractional kinetic equation, generalized K_4 - function, generalized M – Series, Generalized Mittag – Leffler function, Laplace transform operator

1 Introduction and Preliminaries

Fractional kinetic equation have gained importance due to their occurrence in science and engineering, the generalized fractional kinetic equation in term of Mittag – Leffler function studied by sexcena, Mathai and Haubold [16], they introduced the solution of the generalized fractional kinetic equation associated with the generalized Mittag – Leffler function. For more result one can refer to the work of Saichev and Zaslavsky [11], Sexcena et al [15, 17] Zaslavsky [19] and Sexcena and Kalla [14].

Chaurasia and Kumar derived the solution of generalized fractional kinetic equation involving the M – Series [], the result obtained is quite general in nature and capable of yielding a very large number of results.

On the other hand Gupta and Parihar [6] investigated the solution of generalized fractional kinetic equation in term of the K_4 - function and give more precise result of the work of Chaurasia and Pandey [2] and Sharma [18].

In this paper we will derive the solution of the generalized fractional kinetic equation in terms of the generalized Mittag– Leffler function introduced by Salim and Faraj [12], the generalized M – Series and the generalized K_4 - function introduced by the authors in a different papers [4, 5] as

$$E_{\alpha,\beta,q}^{\gamma,\delta,p}(z) = \sum_{k=0}^{\infty} \frac{(\gamma)_{pk}}{\Gamma(\alpha k + \beta)} \frac{z^k}{(\delta)_{qk}} \quad (1.1)$$

$$M_{p,q,m,n}^{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{z^k}{\Gamma(\alpha k + \beta)} \quad (1.2)$$

and

$$K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,c),(p,q)}(z) = \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(\gamma)_k (a)^k (z-c)^{(k+\gamma)\alpha-\beta-1}}{k! \Gamma((k+\gamma)\alpha-\beta)} \quad (1.3)$$

During this paper one can use the following:

- Generalized Riemman – Liouville operator of fractional calculus [8, 10]

$${}_c D_z^{-\nu} f(z) = \frac{1}{\Gamma(\nu)} \int_c^z (z-u)^{\nu-1} f(u) du \quad \text{Re}(\nu) > 0, \quad z > c \quad (1.4)$$

and

$${}_c D_z^{\mu} f(z) = \frac{d^k}{dz^k} ({}_c D_z^{\mu-k} f(z)) \quad \text{Re}(\mu) > 0, \quad k - \mu > 0 \quad (1.5)$$

- Def of Laplace transform and convolution theorem [10]

$$\mathcal{L}\{f(t); p\} = \int_0^{\infty} e^{-pt} f(t) dt \quad (1.6)$$

$$\mathcal{L}\{f * g\}(p) = \mathcal{L}\left\{\int_0^t f(t-\xi) g(\xi) d\xi\right\} = \mathcal{L}\{f(p)\} \cdot \mathcal{L}\{g(p)\} \quad (1.7)$$

and

$$\mathcal{L} \left\{ \frac{t^{n-1}}{\Gamma(n)}; \rho \right\} = \frac{1}{p^n} \quad (1.8)$$

also from [8] we have

$$\mathcal{L} \left\{ {}_0D_t^{-\nu} N(t); \rho \right\} = \rho^{-\nu} N(\rho) \quad (1.9)$$

The following two relation is needed

$$(1-x)^{-a} = \sum_{k=0}^{\infty} \frac{(a)_k (x)^k}{k!} \quad (1.10)$$

$$\sum_{r=0}^n \binom{n}{r} (x)^r = (1+x)^n \quad (1.11)$$

2 Solving General Fractional Kinetic Equation

On integrating the standard kinetic equation

$$\frac{d}{dt} N_i(t) = -c_i N_i(t) \quad c_i > 0 \quad (2.1)$$

Haubold and Mathai [7] derived that

$$N_i(t) - N_i(0) = -c_i {}_0D_t^{-1} N_i(t) \quad (2.2)$$

Where ${}_0D_t^{-1}$ is the standard Riemann integral operator, N_i is number density of species i , which is a function of time t and $N_i(0) = N_0$ is the number density of that species of time $t=0$. By dropping the index i and replacing the Riemann integral ${}_0D_t^{-1}$ operator by the fractional Riemann – Liouville operator ${}_0D_t^{-\nu}$, the kinetic equation (2.2) reduces to the generalized fractional kinetic equation

$$N(t) - N_0 = -c^\nu {}_0D_t^{-\nu} N(t) \quad (2.3)$$

Taking Laplace transform operator for both side we get

$$N(p) = \frac{N_0}{1 + c^\nu p^{-\nu}}$$

$$\text{hence} \quad N(p) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k (c^\nu)^k}{p^{\nu k}}$$

taking Laplace inverse, we get

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k (ct)^{vk}}{\Gamma(vk+1)} \quad (2.4)$$

Lemma 2.1:

The fractional kinetic equation

$$N(t) - N_0 t^{\mu-1} = -c^v {}_0D_t^{-v} N(t) \quad (2.5)$$

have the solution

$$N(t) = N_0 \Gamma(\mu) t^{\mu-1} M_{0,0}^{v,\mu}(-, -, -c^v t^v) \quad (2.6)$$

We will using the illustrated technique above to solve different type of the generalized fractional kinetic equation which contain our special functions in the kernel.

Theorem 2.2:

The general fractional kinetic equation:

$$N(t) - N_0 t^{\beta-1} M_{p,q,m,n}^{\alpha,\beta}(at^\alpha) = -c^v {}_0D_t^{-v} N(t) \quad (2.7)$$

have the solution

$$N(t) - N_0 t^{\beta-1} \sum_{r=0}^{\infty} (-1)^r (ct)^{vr} M_{p,q,m,n}^{\alpha,\beta+vr}(at^\alpha) \quad (2.8)$$

Proof:

Beginning with (2.7) and taking Laplace transform for both sides

$$\mathcal{L}\{N(t); p\} - \mathcal{L}\left\{N_0 t^{\beta-1} M_{p,q,m,n}^{\alpha,\beta}(at^\alpha); p\right\} = \mathcal{L}\{-c^v {}_0D_t^{-v} N(t); p\}$$

Hence by applying (1.9)

$$N(p) - N_0 \mathcal{L}\left\{\sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(a)^k t^{\alpha k + \beta - 1}}{\Gamma(\alpha k + \beta)}\right\} = -c^v p^{-v} N(p)$$

$$N(p) [1 + c^v p^{-v}] = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(a)^k}{\Gamma(\alpha k + \beta)} \mathcal{L}\left\{\frac{t^{\alpha k + \beta - 1}}{\Gamma(\alpha k + \beta)}\right\}$$

and by applying formula (1.8) we get

$$N(p) = \frac{N_0}{(1+c^v p^{-v})} \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (a)^k}{(b_1)_{kn}, \dots, (b_q)_{kn} p^{\alpha k + \beta}}$$

using (1.10) yields,

$$N(p) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^v)^r \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (a)^k}{(b_1)_{kn}, \dots, (b_q)_{kn} p^{\alpha k + \beta + vr}}$$

and now taking Laplace inverse to the last equation

$$\mathcal{L}^{-1}\{N(p)\} = N_0 \sum_{r=0}^{\infty} (-1)^r (c^v)^r \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (a)^k}{(b_1)_{kn}, \dots, (b_q)_{kn}} \mathcal{L}^{-1}\left\{\frac{1}{p^{\alpha k + \beta + vr}}\right\}$$

we get

$$N(t) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^v)^r \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (a)^k}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{t^{\alpha k + \beta + vr - 1}}{\Gamma(\alpha k + \beta + vr)} \quad (2.9)$$

Finally the solution becomes

$$N(t) = N_0 t^{\beta-1} \sum_{r=0}^{\infty} (-1)^r (ct)^{vr} M_{p,q,m,n}^{\alpha, \beta+vr}(at^{\alpha})$$

Remark:

The result obtained coincides with the result obtained by the authors [see 5], for the solution by using alternating method to see that we have from (2.9)

$$\begin{aligned} N(t) &= N_0 \sum_{r=0}^{\infty} (-1)^r (c^v)^r \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (1)_k (a)^k t^{\alpha(k+1) - (\alpha - \beta - vr) - 1}}{(b_1)_{kn}, \dots, (b_q)_{kn} k! \Gamma(\alpha(k+1) - (\alpha - \beta - vr))} \\ &= N_0 \sum_{r=0}^{\infty} (-1)^r (c^v)^r K_{4(m,n)}^{(\alpha, \alpha - \beta - vr, 1), (a, 0), (p, q)}(at^{\alpha}) \end{aligned} \quad (2.10)$$

Theorem 2.3:

The solution of the general fractional kinetic equation

$$N(t) - N_0 t^{\beta-1} E_{\alpha, \beta, n}^{a, b, m}(at^{\alpha}) = -c^v {}_0D_t^{-v} N(t) \quad (2.11)$$

is given by

$$N(t) = N_0 t^{\beta-1} \sum_{r=0}^{\infty} (-1)^r (ct)^{vr} E_{\alpha, \beta+vr, n}^{a, b, m}(at^\alpha) \quad (2.12)$$

Proof:

Taking Laplace transform for both side of (2.11)

$$\mathcal{L}\{N(t); p\} - \mathcal{L}\{N_0 t^{\beta-1} E_{\alpha, \beta, n}^{a, b, m}(at^\alpha); p\} = \mathcal{L}\{-c^\nu {}_0D_t^{-\nu} N(t); p\}$$

we get

$$N(p) - N_0 \sum_{k=0}^{\infty} \frac{(a)_{km} a^k}{(b)_{kn}} \mathcal{L}\left\{\frac{t^{\alpha k + \beta - 1}}{\Gamma(\alpha k + \beta)}\right\} = -c^\nu \rho^{-\nu} N(p)$$

Or

$$N(p) = \frac{1}{(1 + c^\nu \rho^{-\nu})} \sum_{k=0}^{\infty} \frac{(a)_{km} a^k}{(b)_{kn}} p^{\alpha k + \beta}$$

making use of (1.10), $N(p)$ can be written as

$$N(p) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r \sum_{k=0}^{\infty} \frac{(a)_{km} a^k}{(b)_{kn}} p^{(\alpha k + \beta + vr)}$$

and by taking Laplace inverse to the last equation,

$$\mathcal{L}^{-1}\{N(p)\} = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r \sum_{k=0}^{\infty} \frac{(a)_{km} a^k}{(b)_{kn}} \mathcal{L}^{-1}\left\{\frac{1}{p^{\alpha k + \beta + vr}}\right\}$$

we get

$$N(t) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r \sum_{k=0}^{\infty} \frac{(a)_{km} a^k}{(b)_{kn}} \frac{1}{\Gamma(\alpha k + \beta + vr)} t^{\alpha k + \beta + vr - 1}$$

Therefore $N(t)$ becomes

$$N(t) = N_0 t^{\beta-1} \sum_{r=0}^{\infty} (-1)^r (ct)^{vr} E_{\alpha, \beta+vr, n}^{a, b, m}(at^\alpha)$$

Another form of the solution can be written as

$$N(t) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r K_{4(m, n)}^{(\alpha, \alpha - \beta - vr, 1), (a, 0), (1, 1)}(t) \quad (2.13)$$

in which given by the authors in another paper [5].

Theorem 2.4:

The general fractional Kinetic equation

$$N(t) - N_0 K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,c),(p,q)}(t) = -c^\nu {}_0D_t^{-\nu} N(t) \quad (2.14)$$

has the solution given by

$$N(t) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r K_{4(m,n)}^{(\alpha,\beta-\nu r,\gamma),(a,c),(p,q)}(t) \quad (2.15)$$

Proof:

As before in theorem (2.2), (2.3) and by taking Laplace transform of (2.14), we get

$$N(p)(1 + c^\nu p^{-\nu}) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(a)^k (\gamma)_k}{k!} \cdot \mathcal{L} \left\{ \frac{(t-c)^{(k+\gamma)\alpha-\beta-1}}{\Gamma((k+\gamma)\alpha-\beta)} \right\}$$

Or

$$N(p)(1 + c^\nu p^{-\nu}) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(a)^k (\gamma)_k}{k!} \cdot \frac{1}{p^{\alpha(k+\gamma)-\beta}}$$

Hence and by using (1.10) we can rewrite $N(p)$ as

$$N(p) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(a)^k (\gamma)_k}{k!} p^{\alpha(k+\gamma)-\beta+\nu r}.$$

Again by applying Laplace inverse to both sides of the last equation we get

$$N(p) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \frac{(a)^k (\gamma)_k}{k!} \cdot \left\{ \frac{(t-c)^{(k+\gamma)\alpha-\beta+\nu r-1}}{\Gamma((k+\gamma)\alpha-\beta+\nu r)} \right\}$$

Finally $N(t)$ can be written as

$$N(t) = N_0 \sum_{r=0}^{\infty} (-1)^r (c^\nu)^r K_{4(m,n)}^{(\alpha,\beta-\nu r,\gamma),(a,c),(p,q)}(t)$$

which ends the proof

Corollary 2.5:

The general fractional kinetic equation

$$N(t) - N_0 K_{4(m,n)}^{(\nu,\mu,\delta)(-c^\nu,0)(p,q)}(t) = -c^\nu {}_0D_t^{-\nu} N(t) \quad (2.16)$$

has the solution given by

$$N(t) = N_0 K_{4(m,n)}^{(v, \mu+v, \delta+1); (-c^v, 0); (p, q)}(t) \quad (2.17)$$

Proof:

Beginning with (2.16) and taking Laplace transform to both sides,

$$\mathcal{L}\{N(t); p\} - \mathcal{L}\left\{N_0 K_{4(m,n)}^{(v, \mu, \delta); (-c^v, 0); (p, q)}(t); p\right\} = \mathcal{L}\left\{-c^v {}_0D_t^{-v} N(t); p\right\}$$

yields

$$N(p)(1+c^v p^{-v}) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (-c^v)^k (\delta)_k}{(b_1)_{kn}, \dots, (b_q)_{kn} k!} \mathcal{L}\left\{\frac{t^{(k+\delta)v-\mu-1}}{\Gamma((k+\delta)v-\mu)}\right\}$$

Or

$$N(p)(1+c^v p^{-v}) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (-c^v)^k (\delta)_k}{(b_1)_{kn}, \dots, (b_q)_{kn} k! p^{(k+\delta)v-\mu}}$$

and by using (1.10), we get

$$N(p)(1+c^v p^{-v}) = \frac{N_0 p^{\mu-\delta v}}{(1+c^v p^{-v})^\delta} \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}}$$

Or

$$N(p) = \frac{N_0 p^{\mu-\delta v}}{(1+c^v p^{-v})^{\delta+1}} \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \quad (2.18)$$

and now taking Laplace inverse of (2.18), and again applying (1.10) we get

$$N(t) = N_0 \mathcal{L}^{-1}\left\{\sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (\delta+1)_k (-c^v)^k}{(b_1)_{kn}, \dots, (b_q)_{kn} k! p^{v(k+\delta)-\mu}}\right\}$$

Hence

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (\delta+1)_k (-c^v)^k t^{v(k+\delta)-\mu-1}}{(b_1)_{kn}, \dots, (b_q)_{kn} k! \Gamma((k+\delta)v-\mu)}$$

One can rewrite the last equation as

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (\delta+1)_k (-c^v)^k}{(b_1)_{kn}, \dots, (b_q)_{kn} k!} \frac{t^{v(k+\delta+1)-(\mu+v)-1}}{\Gamma((k+\delta+1)v-(\mu+v))}$$

Equivalently

$$N(t) = N_0 K_{4(m,n)}^{(v, \mu+v, \delta+1); (-c^v, 0); (p, q)}(t)$$

A new result related a different general fractional Kinetic equation is given in the next theorem

Theorem 2.6:

The general fractional Kinetic equation

$$N(t) - N_0 K_{4(m,n)}^{(v, \mu, \delta); (-c^v, 0); (p, q)}(t) = - \sum_{r=1}^n \binom{n}{r} c^{rv} {}_0D_t^{-v} N(t) \quad (2.19)$$

has the solution given by

$$N(t) = N_0 K_{4(m,n)}^{(v, \mu + nv, \delta + n); (-c^v, 0); (p, q)}(t) \quad (2.20)$$

Proof:

Beginning with (2.19) and applying Laplace transform to both sides and by using (1.10)

$$\mathcal{L}\{N(t); p\} - \mathcal{L}\left\{N_0 K_{4(m,n)}^{(v, \mu, \delta); (-c^v, 0); (p, q)}(t); p\right\} = \mathcal{L}\left\{- \sum_{r=1}^n \binom{n}{r} c^{rv} {}_0D_t^{-v} N(t)\right\}$$

the result is

$$N(p) \left[1 + \sum_{r=1}^n \binom{n}{r} c^{rv} p^{-rv} \right] = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (-c^v)^k (\delta)_k}{(b_1)_{kn}, \dots, (b_q)_{kn} k! p^{(k+\delta)v - \mu}}$$

Or

$$N(p) \left(\sum_{r=0}^n \binom{n}{r} (c^v p^{-v})^r \right) = \frac{N_0 p^{\mu - \delta v}}{(1 + c^v p^{-v})^{\delta}} \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}}$$

Making use of (1.10) and (1.11), we get

$$N(p) = \frac{N_0 p^{\mu - \delta v}}{(1 + c^v p^{-v})^{\delta + n}} \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn}} \quad (2.21)$$

Now taking Laplace inverse transform of (2.21), yields

$$N(t) = N_0 \mathcal{L}^{-1} \left\{ \sum_{k=0}^{\infty} \frac{(\delta + n)_k (-c^v)^k (a_1)_{km}, \dots, (a_p)_{km}}{(b_1)_{kn}, \dots, (b_q)_{kn} k! p^{v(k+\delta) - \mu}} \right\}$$

Hence $N(t)$ become

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (\delta + n)_k (-c^v)^k t^{(k+\delta)v - \mu - 1}}{(b_1)_{kn}, \dots, (b_q)_{kn} k! \Gamma((k + \delta)v - \mu)}$$

in which the last equation can be written as

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(a_1)_{km}, \dots, (a_p)_{km} (\delta+n)_k (-c^v)^k t^{(k+\delta+n)v - (\mu+nv)-1}}{(b_1)_{kn}, \dots, (b_q)_{kn} k! \Gamma((k+\delta+n)v - (\mu+nv))}$$

Or

$$N(t) = N_0 K_{4(m,n)}^{(v, \mu+nv, \delta+n); (-c^v, 0); (p,q)}(t)$$

which gives the result.

Acknowledgements. The authors wish to thank the anonymous referee for valuable comments and suggestions.

References

- (1) V. Chaurasia, and D. Kumar, On the solutions of generalized kinetic equation, *Adv. Studies theor. Phys.*, vol.4, no.16, (2010) 773 - 780.
- (2) V. Chaurasia, and S. Pandey, On the new computable solutions of generalized fractional kinetic equations involving the generalized function for the fractional calculus and related functions, *Astrophysics and space science*, (2008) 317, 213 - 219. <http://dx.doi.org/10.1007/s10509-008-9880-x>
- (3) A. Erdelyi, W. Magnus, F. Oberhettinger, and F. Tricomi, *Higher transcendental functions*, vol.1, McGraw Hill, New York – Toronto – London (1953).
- (4) A. Faraj, T. Salim, S. Sadek, and J. Ismail, A generalization of M-Series and integral operator associated with fractional calculus (under publication).
- (5) A. Faraj, T. Salim, S. Sadek, and J. Ismail, Generalized K_4 -function and its application in solving kinetic equation of fractional order (under publication).
- (6) A. Gupta, and C. Parihar, On solutions of generalized kinetic equations of fractional order, *Bol. Soc. Paran. Math.* Vo. 32, 1, (2014) 1983 - 191.
- (7) H. Haubold, and A. Mathai, The fractional kinetic equation and thermonuclear functions, *Astrophysics and Space – Sciences* (2000) 327, 53 - 63. <http://dx.doi.org/10.1023/a:1002695807970>
- (8) C. Lorenzo, and T. Hartley, Generalized functions for the fractional calculus, *NASA/ Tp* (2000) 2000 - 210361.

- (9) K. Miller, and B. Ross, An introduction to the fractional calculus and fractional differential equations, John Wiley and sons, New York (1993).
- (10) K. Oldham, and J. Spanier, The fractional calculus, theory and applications of differentiation and integration to arbitrary order, Academic Press, New York, (1974).
- (11) A. Saichev, and G. Zaslavsky, Fractional kinetic equations, solutions and applications, Chaos, (1997) 7, 753 - 784. <http://dx.doi.org/10.1063/1.166272>
- (12) T. Salim, and A. Faraj, A generalization of Mittag – Leffler type function and integral operator associated with fractional calculus, Jou. Frac. Cal. appl. vol.3, No.5, (2012) 1 - 13.
- (13) S. Samko, A. Kilbas, and O. Marichev, Fractional integrals and derivatives, Theory and applications, Gordon and Breach Sci. Publ., New York, (1993).
- (14) R. Sexena, and S. Kalla, On the solution of certain fractional kinetic equations, Appl. Math. Comput, (2008) 199, 504 - 511. <http://dx.doi.org/10.1016/j.amc.2007.10.005>
- (15) R. Sexena, A. Mathai, and H. Haubold, On fractional kinetic equations, Astrophysics and Space - Sci, (2002) 282, 281 - 287.
- (16) R. Sexena, A. Mathai A. and H. Haubold, On generalized fractional kinetic equations, physica A, (2004) 334, 657 - 664. <http://dx.doi.org/10.1016/j.physa.2004.06.048>
- (17) R. Sexena, A. Mathai, and H. Haubold, Unified fractional kinetic equation and a fractional diffusion equation, Astrophysics and Space - Science, (2002) 290, 299 - 310.
- (18) K. Sharma, On the solutions of generalized fractional kinetic equation involving the functions for the fractional calculus, Global Journal of science frontier research vo. 11(2011), 64 - 68.
- (19) G. Zaslavsky, Fractional kinetic equation for Hamiltonian Chaos, Physica D, (1994) 76, 110 - 122. [http://dx.doi.org/10.1016/0167-2789\(94\)90254-2](http://dx.doi.org/10.1016/0167-2789(94)90254-2)

Received: September 14, 2014; Published: November 12, 2014