International Mathematical Forum, Vol. 12, 2017, no. 13, 619 - 628
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/imf.2017.7437

On Left Derivations of Ranked Bigroupoids

Didem Sürgevil Uzay and Alev Firat

Ege University, Institute of Science
Department of Mathematics
35100-Izmir, Turkey

Copyright © 2017 Didem Sürgevil Uzay and Alev Firat. This article is distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

In this paper, we consider regular $(X, *, w)$ self-left derivation and d-invariant on ranked $*$-subsystems of X.Finally,we introduce the notion of $(X, *, \omega)$-left derivation of ranked bigroupoids and discuss some related properties.

Mathematics Subject Classification: 06F35, 03G25

Keywords: ranked bigroupoids, $(X, *, \omega)$ self-left derivation,regular,ranked d-invariant,$*$-subsystems, $(X, *, \omega)$-left derivation

1. Introduction

In the theory of rings and near rings,the properties of derivations are important.In [8], Jun and Xin applied the notion of derivations in rings and near-rings theory to BCI-algebras and also introduced a regular derivation in BCI- algebras .In [11],Zhan and Liu introduced f-derivations that generalized derivation in BCI-algebras and then investigated a regular left derivation.In [1],Abujabal and Al-Shehri introduced the notion of left derivation and gave a condition for left derivation to be regular,and they gave a characterization of a left derivation of a semisimple BCI-algebras.In [2],Alp and Fırat introduced the notion of right derivation of a weak BCC-algebras and investigated some related properties.In [3],Alshehri,Kim and Neggers introduced the notion of ranked bigroupoids and defined $(X, *, \omega)$-self-(co) derivations,rankomorphisms,$(X, *, \omega)$-scalars and $(X, *, \omega)$-derivation for ranked bigroupoids and considered some properties of these.In [6],Jun,Kim and Roh,further properties on $(X, *, \omega)$-self-(co) derivations of ranked bigroupoids are investigated and conditions for an

$(X, *, \omega)$-self-(co) derivation to be regular are provided.In [7],Jun,Lee and Park introduced the notion of generalized coderivation of ranked bigroupoids and the notion of $(X, *, \omega)$-derivation of ranked bigroupoids and discuss some related properties.
In this paper,we introduce the notion of $(X, *, \omega)$ self-left derivation and $(X, *, \omega)$-left derivation on ranked bigroupoids,and investigate some related properties.

2. Preliminaries

In a nonempty set X with a constant 0 and a binary operation $*$, we consider the following axioms for all $x, y, z \in X$:

(a1) $((x * y) * (x * z)) * (z * y) = 0$,
(a2) $((x * (x * y)) * y = 0$,
(a3) $x * x = 0$,
(a4) $x * y = 0$ and $y * x = 0$ imply $x = y$,
(b1) $x * 0 = x$,
(b2) $(x * y) * z = (x * z) * y$,
(b3) $((x * z) * (y * z)) * (x * y) = 0$,
(b4) $x * (x * (x * y)) = x * y$.

If X satisfies axioms (a1),(a2),(a3) and (a4),then we say that $(X, *, 0)$ is a BCI-algebra.Note that a BCI-algebra $(X, *, 0)$ satisfies conditions $(b1)$, $(b2)$, $(b3)$ and $(b4)$. In a p-semisimple BCI-algebra X,the following hold:

(b5) $(x * z) * (y * z) = x * y$,
(b6) $0 * (0 * x) = x$,
(b7) $a * (a * x) = x, \ \forall a \in X$,
(b8) $x * a = x * b$ imply $a = b$,
(b9) $(x * y) * (z * w) = (x * z) * (y * w), \ \forall w \in X$.

Definition 2.1. ([3]) Let $(X, *, \omega)$ be a ranked bigroupoid. A map $d : X \to X$ is said to be an $(X, *, \omega)$-self-derivation if for all $x, y \in X$,

$$d(x * y) = (d(x) * y)\omega(x * d(y)).$$

Definition 2.2. ([3]) Let $(X, *, \omega)$ be a ranked bigroupoid. A map $d : X \to X$ is said to be an $(X, *, \omega)$-self-coderivation if for all $x, y \in X$,

$$d(x * y) = (x * d(y))\omega(d(x) * y).$$

Definition 2.3. ([3]) Let $(X, *, \omega)$ be a ranked bigroupoid. A map $d : X \to X$ is said to be an abelian-$(X, *, \omega)$-self-derivation if it is both an $(X, *, \omega)$-self-derivation and an $(X, *, \omega)$-self-coderivation.

3. Self-Left Derivations on Ranked Bigroupoids

Definition 3.1. Let $(X, *, \omega)$ be a ranked bigroupoid. A map $d : X \to X$ is said to be $(X, *, \omega)$ self-left derivation if it satisfies the identity,

$$d(x * y) = (x * d(y))\omega(y * d(x)) \quad for \ all \ x, y \in X.$$

Example 3.1. *Let $X=\{0,a,b\}$ be a ranked bigroupoid with Cayley table as follows.*

$*$	0	a	b
0	0	0	b
a	a	0	b
b	b	b	0

ω	0	a	b
0	0	0	0
a	0	a	0
b	b	b	0

Define a map $d : X \to X$ for all $x \in X$ by

$$d(x) = \begin{cases} b, & x = 0, a \\ 0, & x = b \end{cases}$$

Then It is easily checked that d is an $(X, *, \omega)$ self-left derivation.

Proposition 3.2. *Let $(X, *, \omega)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation ω is defined by $x\omega y = y * (y * x)$ for all $x, y \in X$.*

(1) *If X satisfies $(a3)$,$(b1)$ and a map $d : X \to X$ is an $(X, *, \omega)$ self-left derivation, then $x * d(x) = y * d(y)$ for all $x, y \in X$.*
(2) *If X satisfies $(a2), (a3), (a4)$,$(b1)$ and a map $d : X \to X$ is an $(X, *, \omega)$ self-left derivation,then $d(x) = d(x)\omega x$ for all $x, y \in X$.*

Proof.

(1) Let $x, y \in X$.Using $(a3)$ and $(b1)$,we have

$$\begin{aligned} d(0) &= d(x * x) \\ &= (x * d(x))\omega(x * d(x)) \\ &= (x * d(x)) * ((x * d(x)) * (x * d(x)) \\ &= (x * d(x)) * 0 \\ &= (x * d(x)). \qquad \textbf{(3.1)} \end{aligned}$$

Similarly,we get

$$\begin{aligned} d(0) &= d(y*y) \\ &= (y*d(y))\omega(y*d(y)) \\ &= (y*d(y))*((y*d(y))*(y*d(y)) \\ &= (y*d(y))*0 \\ &= (y*d(y)). \qquad \textbf{(3.2)} \end{aligned}$$

Hence, we find $x*d(x) = y*d(y)$.

(2) Let $x \in X$.Using $(b1)$ and (1),we have

$$\begin{aligned} d(x) &= d(x*0) \\ &= (x*d(0))\omega(0*d(x)) \\ &= (0*d(x))*((0*d(x))*(x*d(0)) \\ &= (0*d(x))*((0*d(x))*(x*(x*d(x))) \\ &= (0*d(x))*((0*d(x))*(d(x)\omega x)). \end{aligned}$$

Hence,we get $d(x)*(d(x)\omega x) = 0$. Therefore It follows from $(a2)$ that
$(d(x)\omega x)*d(x) = x*(x*d(x))*d(x) = 0$ and so we can write $d(x) = d(x)\omega x$ by $(a4)$. □

Corollary 3.3. *Let $(X,*,\omega)$ be a ranked bigroupoid in which $(X,*,0)$ is a BCI-algebra and the minor operation ω is defined by $x\omega y = y*(y*x)$ for all $x,y \in X$.If a map $d : X \to X$ is an $(X,*,\omega)$ self-left derivation,then the following hold for all $x,y \in X$:*

(1) $x*d(x) = y*d(y)$
(2) $d(x) = d(x)\omega x$.

Proposition 3.4. *Let $(X,*,\omega)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation ω is defined by $x\omega y = y*(y*x)$ for all $x,y \in X$ and X satisfies $(a2),(a3),(a4),(b1)$.Then $d(x) = x$ for all $x \in X$ if and only if $d(0) = 0$.*

Proof. Let $d(x) = x$ for all $x \in X$.So It is clear that $d(0) = 0$.Conversely,Let $d(0) = 0$ and $x \in X$.Then using Proposition 3.2.(2), (3.1) and $(b1)$,we find

$$\begin{aligned} d(x) &= d(x)\omega x \\ &= x*(x*d(x)) \\ &= x*d(0) \\ &= x*0 = x. \ \square \end{aligned}$$

Corollary 3.5. *Let $(X,*,\omega)$ be a ranked bigroupoid in which $(X,*,0)$ is a BCI-algebra and the minor operation ω is defined by $x\omega y = y*(y*x)$ for all $x,y \in X$.*
If a map $d : X \to X$ is an $(X,*,\omega)$ self-left derivation,then $d(x) = x$ for all $x \in X$ if and only if $d(0) = 0$.

Definition 3.6. (**[6]**)Let $(X, *, \omega)$ be a ranked bigroupoid with distinguished element 0.A self map d of $(X, *, \omega)$ is said to be regular if $d(0) = 0$.

Remark 3.7. Proposition 3.4. shows that a regular $(X, *, \omega)$ self-left derivation which satisfies $(a2), (a3), (a4), (b1)$,is the identity map.

Definition 3.8. (**[6]**)Let $(X, *, \omega)$ be a ranked bigroupoid with distinguished element 0.Let d be a self map of $(X, *, \omega)$.A subset A of X is said to be a ranked $*$-subsystem of X if it satisfies the following:

(r1) $0 \in A$,
(r2) $(\forall x, y \in X)(x \in A, y * x \in A \Rightarrow y \in A)$.

Moreover,if a ranked $*$-subsystem A of X satisfies $d(A) \subseteq A$,then it is said that A is ranked d-invariant.

Theorem 3.9. *Left $(X, *, \omega)$ be a ranked bigroupoid with distinguished element 0 in which four axioms $(a2), (a3), (a4), (b1)$ are valid and the minor operation ω is defined by $x\omega y = y * (y * x)$ for all $x, y \in X$.Then an $(X, *, \omega)$ self-left derivation d is regular if and only if every ranked $*$-subsystem of X is ranked d-invariant.*

Proof. Assume that d is regular and let A be a ranked $*$-subsystem of X.Then From Proposition 3.4. we can write $d(x) = x$ for all $x \in X$.Now let $y \in d(A)$.Then we can write $y = d(x)$ for some $x \in A$ and using $(a3)$,we get

$$\begin{aligned} y * x &= d(x) * x \\ &= x * x = 0. \end{aligned}$$

Therefore we find $y * x \in A$ and so we have $y \in A$.Hence we can write $d(A) \subset A$,i.e. A is ranked d-invariant.Conversely,assume that every ranked $*$-subsystem of X is ranked d-invariant.Then particullarly $A = \{0\}$ is a ranked $*$-subsystem of X. Thus we get $d(A) = d(\{0\}) \subseteq 0$ and therefore we have d(0)=0, and so d is regular. □

Corollary 3.10. *Let $(X, *, \omega)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation ω is defined by $x\omega y = y * (y * x)$ for all $x, y \in X$.Then an $(X, *, \omega)$ self-left derivation d is regular if and only if every ranked $*$-subsystem of X is ranked d-invariant.*

Proposition 3.11. *Let $(X, *, \omega)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation ω is defined by $x\omega y = y * (y * x)$ for all $x, y \in X$.*

(1) *If $(X, *, 0)$ satisfies axiom $(b7)$ and a map $d : X \to X$ is an $(X, *, \omega)$ self-left derivation,then $d(x * y) = x * d(y)$ for all $x, y \in X$.*
(2) *If $(X, *, 0)$ satisfies five axioms $(a1), (a3), (b1), (b7), (b8)$ and a map $d : X \to X$ is an $(X, *, \omega)$ self-left derivation,then $d(x) * x = d(y) * y$ for all $x, y \in X$.*

(3) *If* $(X,*,0)$ *satisfies five axioms* $(a1),(a3),(b1),(b7),(b8)$ *and a map* $d:X\to X$ *is an* $(X,*,\omega)$ *self-left derivation,then* $x*d(x)=d(y)*y$ *for all* $x,y\in X$.

Proof.

(1) Let $x,y\in X$.Using $(b7)$,we have

$$\begin{aligned} d(x*y) &= (x*d(y))\omega(y*d(x)) \\ &= ((y*d(x))*((y*d(x)))*(x*d(y))) \\ &= (x*d(y)). \end{aligned}$$

(2) Let $x,y\in X$.Using $(a1)$,we get $(x*y)*(x*d(y))*(d(y)*y)=0$ and $(y*x)*(y*d(x))*(d(x)*x)=0$.So we find

$$((x*y)*(x*d(y)))*(d(y)*y)=((y*x)*(y*d(x)))*(d(x)*x). \quad \textbf{(3.3)}$$

Using Proposition 3.2(1),we have

$$(x*y)*d(x*y)=(y*x)*d(y*x). \quad \textbf{(3.4)}$$

It follows from (1) and (3.4) that $(x*y)*(x*d(y))=(y*x)*(y*d(x))$. (3.3) implies that $d(x)*x=d(y)*y$ by $(b8)$.

(3) Let X satisfies axioms $(a1),(a3),(b1),(b7),(b8)$.Then from (3.1) we can write $d(0)=x*d(x)$ for all $x\in X$.Beside it follows from (2) and $(b1)$ that

$$d(0)=d(0)*0=d(y)*y \ for \ all \ y \ \in X.$$

Hence we can write $x*d(x)=d(y)*y$ for all $x,y\in X$. □

Corollary 3.12. $(X,*,\omega)$ *be a ranked bigroupoid in which* $(X,*,0)$ *is a p-semisimple BCI-algebra and the minor operation w is defined by* $x\omega y=y*(y*x)$ *for all* $x,y\in X$*.If a map* $d:X\to X$ *is an* $(X,*,\omega)$ *self-left derivation,then the following hold for all* $x,y\in X$:

(1) $d(x*y)=x*d(y)$.
(2) $d(x)*x=d(y)*y$.
(3) $x*d(x)=d(y)*y$.

Theorem 3.13. *Let* $(X,*,\omega)$ *be a ranked bigroupoid with distinguished element* 0 *in which axioms* $(a1),(a3),(b1),(b2),(b7),(b8),(b9)$ *are valid and the minor operation* ω *is defined by* $x\omega y=y*(y*x)$ *for all* $x,y\in X$*.A self map* d *of* X *is* $(X,*,\omega)$ *self-left derivation if and only if it is an abelian-* $(X,*,\omega)$*-self-derivation.*

Proof.

Assume that d is an $(X,*,\omega)$ self-left derivation.Then firstly, we show that d is an $(X,*,\omega)$ self-coderivation.Using Proposition 3.11.(1) and $(b7)$,we have

$$\begin{aligned} d(x*y) &= x*d(y) \\ &= (d(x)*y)*((d(x)*y)*(x*d(y))) \\ &= (x*d(y)\omega(d(x)*y). \end{aligned}$$

Hence d is an $(X,*,\omega)$-self-coderivation. On the other hand we show that d is an $(X,*,\omega)$ self-derivation.Using Proposition 3.11.(1),$(b1)$, $(a3)$, (3.1), Proposition 3.11.(3) and $(b9)$, $(b2)$,we have

$$\begin{aligned} d(x*y) &= x*d(y) \\ &= (x*0)*d(y) \\ &= (x*(d(0)*d(0)))*d(y) \\ &= (x*(x*d(x))*(x*d(x)))*d(y) \\ &= (x*(x*d(x))*(d(y)*y))*d(y) \\ &= (x*(x*d(y))*(d(x)*y)))*d(y) \\ &= (x*d(y))*((x*d(y))*(d(x)*y)) \\ &= (d(x)*y))\omega(x*d(y)). \end{aligned}$$

Conversely Let d is an abelian-$(X,*,\omega)$-self-derivation.Then d is an $(X,*,\omega)$-self-coderivation.Now using $(b7)$,we get

$$\begin{aligned} d(x*y) &= (x*d(y))\omega(d(x)*y) \\ &= (d(x)*y)*((d(x)*y)*(x*d(y))) \\ &= (x*d(y)) \\ &= (y*d(x))*((y*d(x))*(x*d(y))) \\ &= (x*d(y))\omega(y*d(x)). \end{aligned}$$

Hence d is an $(X,*,\omega)$ self-left derivation. □

Corollary 3.14. *$(X,*,\omega)$ be a ranked bigroupoid in which $(X,*,0)$ is a p-semisimple BCI-algebra and the minor operation w is defined by $x\omega y = y*(y*x)$ for all $x,y \in X$.A self map d of X is $(X,*,\omega)$ self-left derivation if and only if it is an abelian-$(X,*,\omega)$-self-derivation.*

4. Left Derivations on Ranked Bigroupoids

Definition 4.1. Let $(X,*,\omega)$ and $(Y,\bullet,\psi)$ be ranked bigroupodis.A map $\delta : (X,*,\omega) \to (Y,\bullet,\psi)$ is said to be an $(X,*,\omega)$-left derivation if there exist a rankomorphism (not neccessarily unique) $f : X \to Y$ such that $\delta(x*y) = (f(x)\bullet\delta(y))\psi(f(y)\bullet\delta(x))$ for all $x,y \in X$.

Example 4.1. *Let $X = \{0,a,b,c\}$ be a ranked bigroupoid with Cayley table as follows.*

$*$	0	a	b	c
0	0	0	0	c
a	a	0	a	c
b	b	b	0	c
c	c	c	c	0

ω	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	c	c	c	c

Define a map $d : X \to X$ for all $x \in X$ by

$$d(x) = \begin{cases} c, & x = 0, a, b \\ 0, & x = c \end{cases}$$

and define a rankomorphism for all $x \in X$ by

$$f(x) = \begin{cases} 0, & x = 0, a, b \\ c, & x = c \end{cases}$$

Then It is easily checked that d is an $(X, *, \omega)$-left derivation.

Proposition 4.2. *Let $(X, *, \omega)$ and $(Y, \bullet, \psi)$ be ranked bigroupodis.If $f : (X, *, \omega) \to (Y, \bullet, \psi)$ is a rankomorphism of ranked bigroupoid and $d : X \to X$ is an $(X, *, \omega)$ self-left derivation,then $f \circ d : (X, *, \omega) \to (Y, \bullet, \psi)$ is an $(X, *, \omega)$-left derivation.*

Proof. Let f be a rankomorphism and d be an $(X, *, \omega)$ self-left derivation.Then

$$\begin{aligned}(f \circ d)(x * y) &= f(d(x * y)) \\ &= f((x * d(y)\omega(y * d(x)) \\ &= f(x * d(y))\psi f(y * d(x)) \\ &= (f(x) \bullet f(d(y)))\psi(f(y) \bullet f(d(x))) \\ &= (f(x) \bullet (f \circ d)(y))\psi(f(y) \bullet (f \circ d)(x))\end{aligned}$$

for all $x, y \in X$.So $f \circ d$ is an $(X, *, \omega)$-left derivation. □

Proposition 4.3. *Let $(X, *, \omega)$ and $(Y, \bullet, \psi)$ be ranked bigroupodis.If $f : (X, *, \omega) \to (Y, \bullet, \psi)$ is a rankomorphism of ranked bigroupoid and $d : Y \to Y$ is an $(Y, \bullet, \psi)$ self-left derivation,then $d \circ f : (X, *, \omega) \to (Y, \bullet, \psi)$ is an $(X, *, \omega)$-left derivation.*

Proof. Suppose that f is a rankomorphism and d is an $(Y, \bullet, \psi)$ self-left derivation.

$$\begin{aligned}d(f(x * y)) &= d(f(x) \bullet f(y)) \\ &= (f(x) \bullet d(f(y)))\psi(f(y) \bullet d(f(x))) \\ &= (f(x) \bullet (d \circ f)(y))\psi(f(y) \bullet (d \circ f)(x))\end{aligned}$$

for all $x, y \in X$ Hence $d \circ f$ is an $(X, *, \omega)$-left derivation. □

Proposition 4.4. *Let $(X,*,\omega)$,$(Y,\bullet,\psi)$,$(Z,\bigtriangledown,\varphi)$ be ranked bigroupodis.If $f : (X,*,\omega)\to(Y,\bullet,\psi)$ is a rankomorphism and if $\delta : (Y,\bullet,\psi)\to(Z,\bigtriangledown,\varphi)$ is a $(Y,\bullet,\psi)$-left derivation,for some rankomorphism $g : (Y,\bullet,\psi)\to(Z,\bigtriangledown,\varphi)$,then $\delta\circ f : (X,*,\omega)\to(Z,\bigtriangledown,\varphi)$ is an $(X,*,\omega)$-left derivation.*

Proof. Let f be a rankomorphism and d is a $(Y,\bullet,\psi)$-left derivation.Then

$$\begin{aligned}(\delta\circ f)(x*y) &= \delta(f(x*y))\\ &= \delta(f(x)\bullet f(y))\\ &= (g(f(x))\bigtriangledown\delta(f(y)))\varphi(g(f(y))\bigtriangledown\delta(f(x)))\\ &= ((g\circ f)(x)\bigtriangledown(\delta\circ f)(y))\varphi((g\circ f)(y)\bigtriangledown(\delta\circ f)(x))\end{aligned}$$

Hence $\delta\circ f$ is an $(X,*,\omega)$-left derivation. □

Proposition 4.5. *Let $(X,*,\omega)$,$(Y,\bullet,\psi)$,$(Z,\bigtriangledown,\varphi)$ be ranked bigroupoids.If $f : (X,*,\omega)\to(Y,\bullet,\psi)$ is a rankomorphism and if $\delta : (X,*,\omega)\to(Y,\bullet,\psi)$ is an*
$(X,*,\omega)$-left derivation,for some rankomorphism $g : (Y,\bullet,\psi)\to(Z,\bigtriangledown,\varphi)$,then $g\circ f : (X,*,\omega)\to(Y,\bullet,\psi)$ is an $(X,*,\omega)$-left derivation.

Proof. Let f is a rankomorphism and δ is an $(X,*,\omega)$-left derivation such that $\delta(x*y)=(f(x)\bullet\delta(y))\psi(f(y)\bullet\delta(x))$ for all $x,y\in X$.Then

$$\begin{aligned}(g\circ\delta)(x*y) &= g(\delta(x*y))\\ &= g((f(x)\bullet\delta(y)))\psi(f(y)\bullet\delta(x)))\\ &= ((g\circ f)(x)\bigtriangledown(g\circ\delta)(y))\varphi((g\circ f)(y)\bigtriangledown(g\circ\delta)(x)).\end{aligned}$$

Hence $g\circ\delta$ is an $(X,*,\omega)$-left derivation. □

Proposition 4.6. *Let $(X,*,\omega)$,$(Y,\bullet,\psi)$,$(Z,\bigtriangledown,\varphi)$,$(T,\bigtriangleup,\star)$ be ranked bigroupoids.If $\alpha : (X,*,\omega)\to(Y,\bullet,\psi)$ and $\beta : (Z,\bigtriangledown,\varphi)\to(T,\bigtriangleup,\star)$ are rankomorphisms and $\delta : (Y,\bullet,\psi)\to(Z,\bigtriangledown,\varphi)$ is a $(Y,\bullet,\psi)$-left derivation,then for some rankomorphism $g : (Y,\bullet,\psi)\to(Z,\bigtriangledown,\varphi)$, $\beta\circ\delta\circ\alpha : (X,*,\omega)\to(T,\bigtriangleup,\star)$ is an $(X,*,\omega)$-left derivation.*

Proof. Let α,β be rankomorphisms and δ is a $(Y,\bullet,\psi)$-left derivation.Then

$$\begin{aligned}\beta\circ\delta\circ\alpha(x*y) &= \beta\circ\delta(\alpha(x*y))\\ &= \beta\circ\delta(\alpha(x)\bullet\alpha(y))\\ &= \beta\circ[(g(\alpha(x))\bigtriangledown\delta(\alpha(y)))\varphi(g(\alpha(y))\bigtriangledown\delta(\alpha(x)))]\\ &= \beta\circ[(g\circ\alpha(x)\bigtriangledown\delta\circ\alpha(y))\varphi(g\circ\alpha(y)\bigtriangledown\delta\circ\alpha(x))]\\ &= \beta(g\circ\alpha(x)\bigtriangledown\delta\circ\alpha(y))\star\beta(g\circ\alpha(y)\bigtriangledown\delta\circ\alpha(x))\\ &= \beta((g\circ\alpha)(x)\bigtriangledown(\delta\circ\alpha)(y))\star\beta((g\circ\alpha)(y)\bigtriangledown(\delta\circ\alpha)(x))\\ &= ((\beta\circ g\circ\alpha)(x)\bigtriangleup(\beta\circ\delta\circ\alpha)(y))\star((\beta\circ g\circ\alpha)(y)\bigtriangleup(\beta\circ\delta\circ\alpha)(x)).\end{aligned}$$

Hence $\beta\circ\delta\circ\alpha$ is an $(X,*,\omega)$-left derivation. □

References

[1] H.A.S. Abujabal, N.O. Al-Shehri, On left derivations of BCI-algebras, *Soochow Journal of Mathematics*, **33** (2007), no. 3, 435 − 444.

[2] N. Alp, A. Fırat, On right derivations of weak BCC-algebras, *Ars Combinatoria*, (2013), In press.

[3] N.O. Alshehri, H.S. Kim and J. Neggers, Derivations on ranked bigroupoids, *Applied Mathematics & Information Sciences*, **7** (2013), no. 1, 161 − 166. https://doi.org/10.12785/amis/070119

[4] H.E. Bell, L.C. Kappe, Rings in which derivations satisfy certain algebraic conditions, *Acta Mathematica Hungarica*, **53** (1989), no. 3 − 4, 339 − 346. https://doi.org/10.1007/bf01953371

[5] H.E. Bell, G. Mason, On derivations in near-rings, Chapter in *North-Holland Mathematics Studies*, Amsterdam, The Netherlands, Vol. 137, 1987, 31 − 35. https://doi.org/10.1016/s0304-0208(08)72283-7

[6] Y.B. Jun, H.S. Kim and E.H. Roh, Further results on derivations of ranked bigroupoids, *Journal of Applied Mathematics*, **2012** (2012), 1 − 9. https://doi.org/10.1155/2012/783657

[7] Y.B. Jun, K.J. Lee and C.H. Park, Coderivations of ranked bigroupoids, *Journal of Applied Mathematics*, **2012** (2012), 1 − 8. https://doi.org/10.1155/2012/626781

[8] Y.B. Jun, X.L. Xin, On derivations of BCI-algebras, *Information Sciences*, **159** (2004), no. 3 − 4, 167 − 176. https://doi.org/10.1016/j.ins.2003.03.001

[9] K. Kaya, Prime rings with α-derivations, *Hacettepe Bulletin of Natural Sciences and Engineering*, **16-17** (1987 − 1988), 63 − 71.

[10] E.C. Posner, Derivations in prime rings, *Proceedings of the American Mathematical Society*, **8** (1957), 1093 − 1100. https://doi.org/10.2307/2032686

[11] J. Zhan, Y.L. Liu, On f-derivations of BCI-algebras, *International Journal of Mathematics and Mathematical Sciences*, **2005** (2005), no. 11, 1675 − 1684. https://doi.org/10.1155/ijmms.2005.1675

Received: April 24, 2017; Published: June 1, 2017