Nurse Scheduling at Navrongo War Memorial Hospital in Ghana Using Linear Programming

D. K. Boah

Department of Mathematics
University for Development Studies
Box 24, Navrongo - Ghana

I. K. Adu

Department of Mathematics
Valley View University, Techiman Campus
Box 183, Brong Ahafo - Ghana

P. A. Osei

Department of Mathematics
University for Development Studies
Box 24, Navrongo - Ghana

Copyright © 2014 D. K. Boah, I. K. Adu and P. A. Osei. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

The objective of the paper was to analyse nurse scheduling at Navrongo War Memorial Hospital in Ghana using linear programming and discuss how the hospital could schedule its nurses for better health care delivery. Based on the data collected and available constraints, a linear programming model was formulated for each day and solved using Management Scientist Version 5.0 Software. The study revealed that the minimum number of nurses required to begin morning shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 66, 63, 58, 63, 59, 56 and 49 respectively. Also, the minimum number of nurses required to begin afternoon shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 23, 26, 24, 19, 21, 23 and 24 respectively. Again, the minimum number of nurses required to begin night shifts on Mondays, Tuesdays, Wednesdays,

Thursdays, Fridays, Saturdays and Sundays were 14, 16, 12, 16, 14, 13 and 12 respectively. For effective health care delivery, the optimal minimum number of nurses required to begin all the shifts (R) on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 103, 105, 94, 98, 94, 92 and 85 respectively. It is recommended that measures should be put in place to ensure that the nurses work well and report to duty on time for effective health care delivery. Also, the hospital should design a system that will electronically generate all the duty schedules or rosters without any difficulties and check attendance of all the workers.

Keywords: Nurse Scheduling Problem, Linear Programming, Shift, Optimal solution

INTRODUCTION

The nurse scheduling problem (NSP), also called the nurse rostering problem (NRP) is the operations research problem of finding an optimal way to assign nurses to shifts, typically with a set of hard constraints which all valid solutions must follow, and a set of soft constraints which define the relative quality of valid solutions (Solos et al., 2013). The Nurse scheduling problem is all about assignment of shifts and holidays to nurses. A shift is a person's scheduled period of work. There are two general types of nurse scheduling namely cyclical and non cyclical scheduling (Curtois, 2007). In cyclical scheduling, a single schedule for a fixed planning period is created, and assigned to all employees. The scheduling restarts once the end of the planning period is reached. Non-cyclical scheduling, as the name suggests, is opposite to cyclical scheduling. Each nurse has a schedule which satisfies his/her personal preferences and requests. The nurse scheduling problem is described as finding a schedule that both respects the constraints of the nurses and fulfils the objectives of the hospital. We search for a solution satisfying as many wishes as possible while not compromising the needs of the hospital. There are two types of constraints namely hard constraints and soft constraints. Hard constraints are constraints that render the entire schedule invalid when they fail. They are constraints that cannot be violated. Hard constraints typically include a specification of shifts (e.g. morning, afternoon, and night), that each nurse should not work for more than one shift per day, and that all patients should have nursing coverage (Solos et al., 2013). Differences in qualifications between nurses also create hard constraints (White, 2004). On the other hand, soft constraints do not render the entire schedule invalid when they fail. They can be violated. Soft constraints may include minimum and maximum numbers of shifts assigned to a given nurse in a given week, of hours worked per week, of days worked consecutively, of days off consecutively, and so on (Solos et al., 2013). Solutions to the nurse scheduling problem use a variety of techniques, including both mathematically exact solutions (Goodman et al., 2009) and a variety of heuristic solutions using decomposition, parallel computing (Lagatie et al., 2009), stochastic optimization (Solos et al., 2013), genetic algorithms, colony optimization, simulated annealing, Tabu search (Goodman et al., 2009) and coordinate descent.

A number of papers in the theory and applications of nurse scheduling problem have been reported in the literature. Warner and Prawda (1972) formulated the problem as a Mixed Integer Quadratic Programming problem. A solution to the problem represented a staffing pattern which specified the number of nurses to cover the shifts for six wards. Bailey (1985) presented an approach which combined the problem of shift planning and the assignment of those shifts to employees while considering some basic work pattern constraints. Cheng et al (1997) presented a constrained programming method for solving a week long NSP in a Hong Kong hospital. Bard and Purnomo (2005) combined heuristic and Integer Programming methods to solve a NSP with up to 100 nurses and approximately 13 hard and soft constraints. Metivier et al (2009) proposed a hybrid approach which emphasized the application of soft global constraints. Maenhout and Vanhoucke (2010) presented an exact Branch-and-Price algorithm for nurse scheduling problem incorporating different branching strategies.

The ranks in Ghana nursing are Staff Nurse, Senior Staff Nurse, Nursing officer, Senior Nurse Officer, Principal Nurse Officer, Deputy Director of Nursing service, and Chief Nursing Officer. For many years, scheduling of nursing staff in Ghanaian hospitals has been done intuitively thereby exposing staff to uneven distribution of working hours and inefficient use of expertise. Navrongo War Memorial Hospital is not an exception. This calls for a schedule that addresses these problems. This paper analysed nurse scheduling at Navrongo War Memorial Hospital in Ghana using linear programming and discussed how the hospital could schedule its nurses for better health care delivery.

MATERIALS AND METHODS

The concept of linear programming was used to analyse nurse scheduling at Navrongo War Memorial Hospital. Linear programming (LP) also called linear optimization is a method used to achieve the best outcome (such as maximum profit or least cost) in a mathematical model whose requirements are represented by linear relationships. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. The interested reader is referred to Gartner & Matousek, 2006 and Vanderbei, 2008 for detailed discussions of linear programming. The hospital had 14 units namely Maternity, Male ward, Nursery, Emergency, Pediatric, Out Patient Department (OPD), Child Welfare, Eye Clinic, ENT Clinic, Psychiatry and Neurological, Family Planning, Anti Retroviral Therapy, Post Natal Clinic and Chest Units. Data of duty rosters for all the nurses in the 14 units of the hospital were collected, studied and summarized as shown in Table 1.

Table 1: Summarized duty schedule for nurses at Navrongo War Memorial Hospital

DAY	MORNING	AFTERNOON	NIGHT	DAY-OFF	LEAVE
Monday	66	23	14	13	5
Tuesday	63	26	16	12	5
Wednesday	58	24	12	23	4
Thursday	63	19	16	18	4
Friday	59	21	14	22	4
Saturday	56	23	13	25	4
Sunday	49	24	12	29	4

SOURCE: Navrongo War Memorial Hospital, 2013.

Formulation of the linear programming model

The nurse scheduling at the Navrongo War Memorial Hospital had three main shifts namely the Morning, Afternoon and Night shifts. Nurses on the morning shift started work at 8:00 am and worked for 6 hours until the nurses on the afternoon shift took over from 2:00 pm to 8:00 pm. The nurses on the night shift continued from 8:00 pm to 8:00 am to makes up a complete day. There were also nurses who were either on day-off or on leave.

- **Decision Variables**

Let the decision variables x_1 , x_2 , x_3 , x_4 and x_5 denote the number of nurses beginning the morning shift per day, afternoon shift per day, the night shift per day, day-off shift per day and nurses on leave respectively.

- **Objective Function**

Our objective was to minimize the number of nurses that would be required for good health care delivery in the hospital for each day. Hence, our objective function was as follows:

Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$ where R is the minimum number of nurses needed in the hospital for a particular day.

- **The Constraints**

The least number of nurses required for the various shifts had the following limitations:

$x_1 \geq a$, $x_2 \geq b$, $x_3 \geq c$, $x_4 \leq d$ and $x_5 \leq e$ where a, b, c, d, e are the total number of nurses required for each of the shifts.

Also the total number of nurses beginning day-off and leave must not exceed the total number of nurses beginning the morning, afternoon and night shifts. That is

$$x_4 + x_5 \leq x_1 + x_2 + x_3$$

Finally the total number of nurses beginning the morning and afternoon shifts must exceed the number of nurses beginning the night shift. That is,

$$x_3 \leq x_1 + x_2$$

$x_i \geq 0$ for $i = 1, 2, 3, 4, 5$ where x_i = number of nurses beginning the i^{th} shift per day.

The resulting linear programming model is therefore given as:

$$\text{Minimize } R = x_1 + x_2 + x_3 + x_4 + x_5$$

$$\begin{aligned} \text{Subject to } & x_1 \geq a, \\ & x_2 \geq b, \\ & x_3 \geq c, \\ & x_4 \leq d \\ & x_5 \leq e \\ & x_1 + x_2 + x_3 - x_4 - x_5 \geq 0 \\ & x_1 + x_2 - x_3 \geq 0 \\ & x_i \geq 0 \text{ for } i = 1, 2, 3, 4, 5 \end{aligned}$$

DATA ANALYSIS, RESULTS AND DISCUSSIONS

From the resulting linear programming model and Table 1, the following linear programming models were obtained for each day:

<p>• Monday</p> <p>Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$</p> <p>Subject to</p> $\begin{aligned} x_1 &\geq 66 \\ x_2 &\geq 23 \\ x_3 &\geq 14 \\ x_4 &\leq 13 \\ x_5 &\leq 5 \\ x_1 + x_2 + x_3 - x_4 - x_5 &\geq 0 \\ x_1 + x_2 - x_3 &\geq 0 \\ x_i &\geq 0 \text{ for } i = 1, 2, 3, 4, 5 \end{aligned}$	<p>• Tuesday</p> <p>Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$</p> <p>Subject to</p> $\begin{aligned} x_1 &\geq 63 \\ x_2 &\geq 26 \\ x_3 &\geq 16 \\ x_4 &\leq 12 \\ x_5 &\leq 5 \\ x_1 + x_2 + x_3 - x_4 - x_5 &\geq 0 \\ x_1 + x_2 - x_3 &\geq 0 \\ x_i &\geq 0 \text{ for } i = 1, 2, 3, 4, 5 \end{aligned}$
---	--

<ul style="list-style-type: none"> Wednesday Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$ Subject to $x_1 \geq 58$ $x_2 \geq 24$ $x_3 \geq 12$ $x_4 \leq 23$ $x_5 \leq 4$ $x_1 + x_2 + x_3 - x_4 - x_5 \geq 0$ $x_1 + x_2 - x_3 \geq 0$ $x_i \geq 0$ for $i = 1, 2, 3, 4, 5$ 	<ul style="list-style-type: none"> Thursday Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$ Subject to $x_1 \geq 63$ $x_2 \geq 19$ $x_3 \geq 16$ $x_4 \leq 18$ $x_5 \leq 4$ $x_1 + x_2 + x_3 - x_4 - x_5 \geq 0$ $x_1 + x_2 - x_3 \geq 0$ $x_i \geq 0$ for $i = 1, 2, 3, 4, 5$
<ul style="list-style-type: none"> Friday Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$ Subject to $x_1 \geq 59$ $x_2 \geq 21$ $x_3 \geq 14$ $x_4 \leq 22$ $x_5 \leq 4$ $x_1 + x_2 + x_3 - x_4 - x_5 \geq 0$ $x_1 + x_2 - x_3 \geq 0$ $x_i \geq 0$ for $i = 1, 2, 3, 4, 5$ 	<ul style="list-style-type: none"> Saturday Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$ Subject to $x_1 \geq 56$ $x_2 \geq 23$ $x_3 \geq 13$ $x_4 \leq 25$ $x_5 \leq 4$ $x_1 + x_2 + x_3 - x_4 - x_5 \geq 0$ $x_1 + x_2 - x_3 \geq 0$ $x_i \geq 0$ for $i = 1, 2, 3, 4, 5$

<ul style="list-style-type: none"> Sunday Minimize $R = x_1 + x_2 + x_3 + x_4 + x_5$ Subject to $x_1 \geq 49$ $x_2 \geq 24$ $x_3 \geq 12$ $x_4 \leq 29$ $x_5 \leq 4$ $x_1 + x_2 + x_3 - x_4 - x_5 \geq 0$ $x_1 + x_2 - x_3 \geq 0$ $x_i \geq 0$ for $i = 1, 2, 3, 4, 5$

Solution of the models

All the seven linear programming models were solved using Management Scientist Version 5.0 Software. The results were summarized as given in Table 2 below:

Table 2: Summarized optimal solutions of the linear programming models for the days

	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	S_6	S_7	R
MONDAY	66	23	14	0	0	0	0	0	13	5	103	75	103
TUESDAY	63	26	16	0	0	0	0	0	12	5	105	73	105
WEDNESDAY	58	24	12	0	0	0	0	0	23	4	94	70	94
THURSDAY	63	19	16	0	0	0	0	0	18	5	98	66	98
FRIDAY	59	21	14	0	0	0	0	0	22	4	94	66	94
SATURDAY	56	23	13	0	0	0	0	0	25	4	92	66	92
SUNDAY	49	24	12	0	0	0	0	0	29	4	85	61	85

Source: Solution from Management Scientist Version 5.0 Software.

From table 2, the following deductions were made:

- (i) The minimum number of nurses required to begin morning shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 66, 63, 58, 63, 59, 56 and 49 respectively.
- (ii) The minimum number of nurses required to begin afternoon shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 23, 26, 24, 19, 21, 23 and 24 respectively.
- (iii) The minimum number of nurses required to begin night shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 14, 16, 12, 16, 14, 13 and 12 respectively.
- (iv) For effective health care delivery, the optimal minimum number of nurses required to begin all the shifts (R) on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 103, 105, 94, 98, 94, 92 and 85 respectively.

CONCLUSIONS AND RECOMMENDATIONS

The concept of linear programming was used to analyse nurse scheduling at Navrongo War Memorial Hospital. The hospital had 14 units namely Maternity, Male ward, Nursery, Emergency, Pediatric, Out Patient Department (OPD), Child Welfare, Eye Clinic, ENT Clinic, Psychiatry and Neurological, Family Planning, Anti Retroviral Therapy, Post Natal Clinic and Chest Units. Data of duty rosters for all the nurses in the 14 units of the hospital were collected, studied and summarized. The nurse scheduling at the Navrongo War Memorial Hospital had three main shifts namely the

Morning, Afternoon and Night shifts. There were also nurses who were either on day-off or on leave.

Based on the data collected and available constraints, a linear programming model was formulated for each day and solved using Management Scientist Version 5.0 Software. The minimum number of nurses required to begin morning shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 66, 63, 58, 63, 59, 56 and 49 respectively. The minimum number of nurses required to begin afternoon shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 23, 26, 24, 19, 21, 23 and 24 respectively. The minimum number of nurses required to begin night shifts on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 14, 16, 12, 16, 14, 13 and 12 respectively. For effective health care delivery, the optimal minimum number of nurses required to begin all the shifts (R) on Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays and Sundays were 103, 105, 94, 98, 94, 92 and 85 respectively.

It is recommended that measures should be put in place to ensure that the nurses work well and report to duty on time for effective health care delivery. Also, the hospital should design a system that will electronically generate all the duty schedules or rosters without any difficulties and check attendance of all the workers.

REFERENCES

- [1] J. Bailey, Integrated days off and shift personnel scheduling, *Computing and Industrial Engineering*, 1985, 9(4) 395-404.
- [2] J. F. Bard and H. W. Purnomo, Preference scheduling for nurses using column generation, *European Journal of Operational Research*, 2005, 164(2), 510 -534.
- [3] B. M. W. Cheng, J. H. M. Lee and J. A. C. K. Wu, A nurse rostering system using constraint programming and redundant modelling, *IEEE Transactions on information technology in biomedicine*, 1997, 1(1), 44- 54.
- [4] T. Curtois, Novel Heuristic and Metaheuristic approaches to the Automated Scheduling of Healthcare Personnel, University of Nottingham, 2007
- [5] B. Gärtner and J. Matoušek , *Understanding and Using Linear Programming*, Berlin: Springer, 2006.
- [6] M. D. Goodman, K. A. Dowsland and J. M. Thompson, A Grasp-Knapsack Hybrid for a Nurse-Scheduling Problem, Cardiff: Cardiff University School of Mathematics, 2009, 1–31.

- [7] S. Ioannis, T. Ioannis and B. Grigorios, A Generic Two-Phase Stochastic Variable Neighborhood Approach for Effectively Solving the Nurse Rostering Problem, *Algorithms* (Multidisciplinary Digital Publishing Institute), 6 (2013), 278–308.
- [8] R. Lagatie, S. Haspeslagh and P. De Causmaecker, *Negotiation Protocols for Distributed Nurse Rostering*, Eindhoven University of Technology Department of Computer Science, 2009.
- [9] B. Maenhout and M. Vanhoucke, Branching strategies in a branch-and-price approach for a multiple objective nurse scheduling problem, *Journal of Scheduling*, 2010, 13(1), 77-93.
- [10] J. P. Metivier, P. Boizumault and S. Loudni, *Solving Nurse Rostering Problems Using Soft Global Constraints in Principles and Practice of Constraint Programming*, Lecture Notes in Computer Science, 2009, 73-87.
- [11] W. Paul, *Building Better Nurse Scheduling Algorithms*, *Annals of Operations Research* (arXiv), 128 (2004), 159–177.
- [12] R. J. Vanderbei, *Linear Programming: Foundations and Extensions*, 3rd ed., *International Series in Operations Research & Management Science*, 2008 (114).
- [13] D. M. Warner and J. Prawda, A Mathematical Programming Model for Scheduling Nursing Personnel in a Hospital, *Management Science*, 1972, 19(4), 411-422.

Received: July 9, 2014