A Note on the Extraction of Gravitational Energy

in an External Schwarzschild Metric

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A method is being proposed to take advantage of the laws of general relativity to extract energy from the gravitational field in the solar system environment.

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1. Introduction

The author proposes a new method to extract energy from the Schwarzschild metric of a normal star like body such as the Sun whose existence in current literature is at least not known to him. The method follows from the geodesic equations of motion of finite mass bodies (of course much smaller compared to the gravitational field producing mass) whose motion is strictly confined along the radial coordinate without any \( \theta \) or \( \phi \) variations in its orbit. We start from the equations of motion given in any standard textbook on general relativity such as Weinberg [1] and show that there are some ‘velocities \( \frac{dr}{dt} \)’ where the body is ‘accelerated outward’ in contrast to the usual case where it is ‘accelerated inward’. These ‘velocities’ do not strictly approach the velocity of light and so can be considered within certain degree of approximations to yield ‘kinetic
energies’ using non-relativistic formulas. However because of the curvature introduced into space-time by general relativity the conclusions here are very different from those of Newtonian gravity where all bodies irrespective of their velocities are accelerated inward towards the massive body producing the gravitational field. This important property can be used along with proper techniques to make the body gain kinetic energy while it moves radially outward in the gravitational field as well as when it moves inward and to suitably extract some kinetic energy at a particular point while supplying some kinetic energy at another point to obtain a net extraction of energy from the process. It is possible to show that theoretically more than $10^{10}$ Joules of energy can be extracted by accelerating a 1Kg mass from the solar surface to infinity and then back.

2. Derivation of the formula for net extracted energy

The integrated equations of motion in the equatorial plane of the standard Schwarzschild coordinate system are [1] (see Eq. 8.4.18 and 8.4.19 of this reference)

$$ r^2 \frac{d \phi}{dt} = JB(r) \quad \text{and} \quad A(r) \left( \frac{dr}{dt} \right)^2 + \frac{J^2}{r^2} - \frac{1}{rB(r)} = -E \quad \text{………(1)} $$

where $J$ and $E$ are constants related to the Newtonian concepts of angular momentum and energy per unit mass. For a particle moving along the direction of radial coordinate (no $\phi$ variation) $J = 0$ and for the external Schwarzschild geometry $B(r) = 1/A(r) = 1 - 2GM/r$. Using these in Eq. (1) we get a simplified relation

$$ \left( \frac{dr}{dt} \right)^2 = -EB^3(r) + B^2(r) \quad \text{………(2)} $$

for a radially moving particle. One can differentiate Eq. (2) to show that

$$ \frac{d^2 r}{dt^2} = -\frac{B}{2} \frac{dB}{dr} + \frac{3}{2B} \frac{dB}{dr} \left( \frac{dr}{dt} \right)^2 \quad \text{………(3)} $$

from which for values of $\left( \frac{dr}{dt} \right)^2$ greater than about 0.36 near the solar surface one can show that $\frac{d^2 r}{dt^2}$ is positive or that we have repulsive gravity irrespective of whether $\frac{dr}{dt}$ is negative or positive. For values of $\left( \frac{dr}{dt} \right)^2$ less than about 0.3 we have the usual case of attractive gravity. This gives us an opportunity to eject a particle like a heavy ion radially outward with $\frac{dr}{dt} \approx 0.6$ at a certain initial value of the radial coordinate $r = r_i$ and then
allow it to increase its \( \frac{dr}{dt} \) and hence its ‘kinetic energy’ while it gains height till it reaches a value of the radial coordinate \( r = r' \gg r_i \). At this point a uniform magnetic field may be applied to turn the charge in a circular arc till it is brought into alignment with a radial direction where it can be made to ‘fall’ along the radial direction (\( \theta \) and \( \phi \) are constants). The entire concept is illustrated in fig. 1 where just before the fall begins the particle is at point B. One can use a strong electric field at this point to reduce its

\[
\left( \frac{dr}{dt} \right)^2
\]


to about 0.25 and to somehow store the energy the particle imparts to the electromagnetic field as a consequence of this reduction of its own kinetic energy. After this the ‘free fall’ of the particle will increase its kinetic energy till point A’ where it is again made to move in a circular arc in an applied magnetic field to bring it to point B’ which is the initial point at which it was ejected. Some energy has to be imparted to it at point B’ by an electric field to increase its

\[
\left( \frac{dr}{dt} \right)^2
\]

to 0.36 so that while moving up to point A it again increases its kinetic energy to repeat the cycle all over again. It can be easily shown that the difference in energy that the particle gives to the electromagnetic field at point B and that it receives from a similar electromagnetic field at B’ is always positive since it has increased its \( \left( \frac{dr}{dt} \right)^2 \) in both the paths B’A (while rising) and BA’ (while falling). One can calculate the value of \( E \) during initial ejection radially outward from point B’ at \( r = r_i \) with \( \frac{dr}{dt} = 0.6 \) from Eq. (2) to obtain

\[
E = \frac{(1 - 2GM / r_i)^2 - 0.36}{(1 - 2GM / r_i)^3}
\]

……(4)

where \( GM = 1.475 \) km for Sun. Thus \( \left( \frac{dr}{dt} \right)^2 \) at any value of \( r \) is given by

\[
\left( \frac{dr}{dt} \right)^2 = (1 - E + 2GME / r)(1 - 2GM / r)^2
\]

……(5)

Since the left hand side of this equation is of the order of 0.36 we can use ‘non-relativistic’ formulas to approximately calculate the kinetic energy per unit mass which is \( 0.5 \left( \frac{dr}{dt} \right)^2 \) without making serious errors in the solar system environment.
3. Results and Discussions

If $r_i$ is taken as the value of the solar radius $6.9598 \times 10^5$ km in Eq. (4) we obtain a value of $E$. This when substituted into Eq. (5) yields $\left(\frac{dr}{dt}\right)^2$ at $r = \infty$ to be $0.360000339$ which is greater than $0.6^2 = 0.36$ hence showing a net gain in kinetic energy while moving out from the solar surface to infinity along the path B'A. At point B when its $\left(\frac{dr}{dt}\right)^2$ has been reduced to $0.5^2 = 0.25$ it starts falling towards point A' on the solar surface where it again increases its $\left(\frac{dr}{dt}\right)^2$ to $0.250001060$. At point B' some energy has to be supplied to the particle as already stated to bring its $\left(\frac{dr}{dt}\right)^2$ back to $0.36$ before it starts repeating its upward movement. With the change made into conventional units where the velocity of light is $c = 3 \times 10^8$ m/s instead of being unity we get a net energy extracted at points B and B' together as $0.5(3 \times 10^8)^2[0.360000339-0.25+0.250001060-0.36] = 6.2944 \times 10^{10}$ Joules per kg of matter cycled up and down from the solar surface to infinity and back. As far as conservation of energy is concerned we note that the massive gravitating body in the Schwarzschild metric is essentially static. There must be a mechanism to see how its gravitational field is changing in order to explain the near spontaneous extraction of energy by the method we have outlined here.

References

Figure 1. Schematic conceptualization of the path of a particle for example a heavy ion which can be used to extract energy from the Schwarzschild field of the Sun. The particle moves clockwise along the path B′--A--B--A′-- B′. The straight line sections are those regions where the motion is influenced by gravity. The circular sections under the influence of magnetic fields are needed to change its radial directions and is shown here expanded for the sake of visual clarity.

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