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A Dynamic Portfolio of American Option Using Fuzzy Binomial Method

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Abstract

A Dynamic Portfolio or Dynamic Asset Allocation is a strategy used to determine the proportion of a number of assets, chosen carefully, in order to achieve optimum performance of the portfolio. In this paper, the portfolio consists only Options traded in the financial market. One of the most famous models of option pricing is Binomial Cox-Ross-Rubinstein (CRR) Model. Using Fuzzy Binomial CRR procedure, the price of option is an interval with specific membership degree, by which the investors are allowed to adjust their portfolios. We make a portfolio dynamically adjusted periodically, in which the membership degree of an option price determines decision of buying or selling option in the simulation. The result shows a significant gain of investment only in a short period of time.

Keywords: American Option, Binomial CRR method, Fuzzy Logic

1 Introduction

Traditionally, uncertainty is undesirable in science and should be avoided. But the modern view is in tolerant of uncertainty. Fuzzy system is related to vagueness and because of that fuzzy system is more famous in the east than the west. Noted in [1], it should not be considered as a substitute for the probabilistic approach but rather as a complementary way to describe the model peculiarities. Western culture is tend to be stricter than the east one. Fuzzy numbers approach can be used in finance due to the need for modeling the uncertainty and vagueness. Sumarti et al. [2] discussed the fuzzification the European Option price and develop a portfolio.

An option is a kind of derivative instrument which is widely used in financial world nowadays. The use of option in modern financial markets has been increasing because of its functions as a speculative tool and insurance against uncertainty. Option is a privilege, sold by one party to another that gives the buyer the right, to buy or sell a stock at an agreed-upon price within a certain period or on a specific date. There is an element of uncertainty involved in option pricing. The pricing of options has become a main concern of financial mathematics in consequence of their utility in today's market. By predicting the optimal interval of an option price, investor can generate a larger profit from the options. The most popular approaches to pricing options is the binomial tree methods, which was proposed by Cox, Ross and Rubinstein [3] in diffusion models and extended by Amin [4] to jump-diffusion models. Xu et.al. [5] considered the numerical analysis of finding an Equivalence of the Binomial tree method and an explicit difference scheme, so its optimal error estimation can be found. Kim et.al. [6] showed the uniform convergence of the binomial tree method for European-style and American-style Asian options.

Many mathematical studies make an attempt to figure on the variation of the model to improve the pricing capability. There are a lot of models in option pricing and one of the most famous models in option pricing is Binomial Cox-Ross-Rubinstein (CRR) method. Research in [7, 8, 9, 10, 11] used Binomial CRR in determining the option price.

As in former option pricing method, volatility is assumed to be constant although it is the most controversial variable. Therefore many studies have concentrated on estimating the volatility. The different method used in estimating volatility certainly causes the variation of the price. Binomial model has been considerably used for computing the optimal option price. In this paper we apply fuzzy system in option pricing model, especially for Binomial CRR model conduce to ascertain a belief degree of an option price. Although there are several modifications in Fuzzy Binomial CRR method, the procedure used in this method has the same fundamental rules as the procedure in ordinary Binomial CRR method [10] and [11]. The later papers concerned about implementation fuzzy system for European option pricing while this paper focus on implementation fuzzy system for American option pricing using Binomial CRR model.

By properly using the suitable model, the investor's portfolios will be more reliable. Based on [11], the implementation of fuzzy system for European option pricing has given a good result as a decision maker tool. This paper expands the Fuzzy Binomial CRR method from later papers so this method can be used to get the optimal range for American option prices. We use stock prices and option prices data from Yahoo Finance taken from February 2013 to April 2014. In addition, we took three different types of stock price movements such as bullish, bearing, and sideways. Furthermore, we use AAPL for bullish stock price, we use IBM as bearing stock price, and we consider PEP as sideways stock price.

2 Fuzzy System

Fuzzy logic has many valued logic in which the truth values of variables may be any real number between 0 and 1. The truth values are described as the degree of membership of an element in the fuzzy set whose value lies in the interval [0,1]. Let $\bar{A}$ be a set called a fuzzy number. The degree of the membership function $\mu_{\bar{A}}$ of an element x in $\bar{A}$ is defined as follows:

- If $\mu_{\bar{A}}(x) = 1$ then x is an exact member of $\bar{A}$.
- If $\mu_{\bar{A}}(x) = 0$ then x is not the member of $\bar{A}$.
- If $\mu_{\bar{A}}(x) = \rho$ with $0 \leq \rho \leq 1$ then x is a member of $\bar{A}$ with membership degree ρ.

Fuzzy set membership function can be represented as trapezoidal, triangle, linear, and others.

Arithmetic operations between two fuzzy numbers or between fuzzy and non fuzzy numbers are defined below. Let $\bar{A} = [a^L, a^R]$ and $\bar{B} = [b^L, b^R]$ be fuzzy numbers, k is Real number.

- $\bar{A} + \bar{B} = \bar{C},\ \bar{C} = [a^L + b^L, a^R + b^R]$,
- $\bar{A} - \bar{B} = \bar{C},\ \bar{C} = [a^L - b^R, a^R - b^L]$,
- $\bar{A} \cdot \bar{B} = \bar{C},\ \bar{C} = [min(a^Lb^L, a^Lb^R, a^Rb^L, a^Rb^R),\ max(a^Lb^L, a^Lb^R, a^Rb^L, a^Rb^R)]$,
- $\frac{\bar{A}}{\bar{B}} = \bar{C},\ \bar{C} = \left[min\left(\frac{a^L}{b^L}, \frac{a^L}{b^R}, \frac{a^R}{b^L}, \frac{a^R}{b^R}\right),\ max\left(\frac{a^L}{b^L}, \frac{a^L}{b^R}, \frac{a^R}{b^L}, \frac{a^R}{b^R}\right)\right]$ where $b^L, b^R \neq 0$,
- $k \odot \bar{A} = \bar{D},\ \bar{D} = [k \odot a^L, k \odot a^R]$ where $\odot$ can be replaced by any operator $+, -, \times, \div$.

The degree of the membership function of the results are $\mu_{\bar{C}}(c) = \max(\mu_{\bar{A}}(c), \mu_{\bar{B}}(c)), c \in \bar{C}$, and $\mu_{\bar{D}}(d) = \mu_{\bar{A}}(d), d \in \bar{A}$.

One of the most important concepts of fuzzy sets is the concept of an α-cut and its variant strong α-cut. Given a fuzzy set A defined on X and any $\alpha \in [0,1]$, the α-cut, ${}^{\alpha}A$, and the strong ${}^{\alpha+}A$ are

$$^{\alpha}A = \{x | A(x) \geq \alpha\},$$
$$^{\alpha+}A = \{x | A(x) > \alpha\}.$$

3 Fuzzy Binomial CRR Option Pricing Model

By applying fuzzy system, the binomial tree model is generalized to a fuzzy binomial tree. In [11], σ is fuzzified into:

$$\mu\,(x) = \begin{cases} 0, & \text{if } x < \left(1 - \frac{\rho}{0.9}\right)\sigma \ or\ x \leq \left(1 + \frac{\rho}{0.9}\right)\sigma, \\ \frac{0.9(x-\sigma)}{\rho\sigma}, & \text{if } \left(1 - \frac{\rho}{0.9}\right)\sigma \leq x < \sigma, \\ \frac{0.9(\sigma - x)}{\rho\sigma}, & \text{if } \quad \sigma \leq x < \left(1 + \frac{\rho}{0.9}\right)\sigma. \end{cases}$$

Parameter ρ represents the range of σ values and is determined by stock price historical data. In this paper, we define a fuzzy number $\widetilde{M}$ as form of a closed interval with the $(1-\alpha)(1-\beta)$ confidence as follows.

$$^{\alpha}\widetilde{M} = \left[\frac{s^2}{1 + z_{g(a)}\sqrt{\frac{2}{n-1}}}, \frac{s^2}{1 - z_{g(a)}\sqrt{\frac{2}{n-1}}}\right]$$

where $g(a) = \left(\frac{1}{2} - \frac{\beta}{2}\right)\alpha + \frac{\beta}{2}$, $g: [0,1] \to \left[\frac{\beta}{2}, 0.5\right]$, and $z_{g(a)} = \Phi^{-1}(1 - g(a))$. Here s^2 is the estimator of σ^2, the variance of the stock prices.

In Binomial CRR, there are two movement, up and down. As the impact of the σ^2 fuzzification [11], the up and down movements in Binomial CRR can be replaced each by 3 (three) states, those are respectively uu,um,ud, and dd, dm, du. In formal, they are defined respectively as follows.

$$uu = e^{(1+\rho)\sigma\sqrt{\Delta t}}, \qquad du = e^{-(1-\rho)\sigma\sqrt{\Delta t}},$$
$$um = e^{\sigma\sqrt{\Delta t}}, \qquad dm = e^{-\sigma\sqrt{\Delta t}},$$
$$ud = e^{(1-\rho)\sigma\sqrt{\Delta t}} \qquad dd = e^{-(1+\rho)\sigma\sqrt{\Delta t}}.$$

The membership degree of uu,um,ud and dd, dm, du are calculated respectively as: $\mu_{uu} = \mu_{ud} = \mu_{du} = \mu_{dd} = 0.1$ and $\mu_{um} = \mu_{dm} = 1$. The probabilities for the Binomial CRR are defined as follows.

$$p_1 = \frac{e^{r\Delta t} - dd}{uu - dd}, \qquad p_2 = \frac{e^{r\Delta t} - dm}{um - dm}, \qquad p_1 = \frac{e^{r\Delta t} - du}{ud - du}.$$

With beginning price S_0, it defines the stock price at $t = n$,

$$S_n = uu^i um^j ud^k du^x dm^y dd^z S_0$$

where $i + j + k + x + y + z = n - 1$, all i, j, k, x, y, z are integers. Its membership degree is

$$\mu_{S_n} = \mu_{uu}^i \mu_{um}^j \mu_{ud}^k \mu_{du}^x \mu_{dm}^y \mu_{dd}^z.$$

Using this definition of the stock price and its membership, we can determine the American option valuation for each time step which is similar to the American option pricing for standard non-fuzzy numbers.

4 Dynamic Portfolios

In order to verify fuzzy binomial option pricing method, we make the portfolio consisting options only in which the membership degree of an option price determines buying or selling option. There are three different types of stock price movements used in this portfolio such as bullish, bearish, and sideways (figure 1). There are AAPL as bullish stock price, IBM as bearish stock price, and PEP as sideways stock price according to the historical data from previous period.

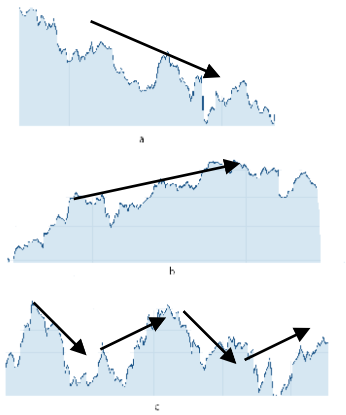

Figure 1. Bullish (upper), bearish (middle) and sideway (lower) types of movement

The transactions days are only 2 April 2014, 7 April 2014, 11 April 2014, 16 April 2014, and 21 April 2014. The option and stock transactions are conducted according to US Market's opening and closing times which are 09.30 AM-16.00 PM EST or 20.30 PM -03.00 AM WIB. Saturday and Sunday are holidays so there are no transactions on these days. It is assume that options sold by the owner are not to be exercised by the buyer to expedite the trading simulation process, and the decisions to buy or sell the stocks are based on membership degree. The selection of the options that will be bought and sold is based on the option prices shown at Yahoo Finance. The option price intervals are calculated by the binomial method which was discussed before.

For example, on 2 April 2014, there were range of option prices with different value of strike prices provided in the market. The market prices for IBM option call and put are not in the interval of calculated fuzzy prices so their membership functions is zero or almost zero. As consequence, we do not buy them. For AAPL with $K = 320, 390, 420, 470, 500, 510, 510, 540$ and PEP with $K = 70, 72.5, 77.5, 80$, we choose the option prices K=470 and K=80 respectively, which have largest value that larger than 0.5. Table 1 shows the example of the choices of these call options.

At starting position at 2 April 2014 the fund is \$50,000. We buy PEP call option with K=80 for 400 unit, AAPL call option with K=470 for 200 unit, PEP put option with K=87 for 300 unit, and AAPL put option with K=570 for 200 units.

	AAPL	AAPL	AAPL
K	**420**	**470**	**500**
Fuzzy option price	122.1455	72.6543	45.8872
	121.7375	71.8484	43.5828
	121.6892	71.6996	43.0840
Market price	**122.50**	**71.90**	**43.15**
Degree	0.0000	0.9424	0.2191

	PEP	PEP	PEP	PEP
K	70.0	72.5	77.5	80.0
Fuzzy option price	12.9161	10.4166	5.4912	3.4594
	12.8946	10.3951	5.4346	3.1954
	12.8917	10.3922	5.4222	3.1534
Market price	**12.85**	**10.30**	**5.47**	**3.25**
Degree	0.0000	0.0000	0.4371	0.8139

Table 1. The comparison of the market and calculated option prices

Prices used to purchase is the ask price, and price used to sell is the bid price. The total spent fund is \$26,853 in this day transaction. The position of this portfolio is evaluated in the next trading day, 7 April 2014.

In 7 April 2014, there were several transactions:

1. Based on the greatest membership degree, we buy PEP call option with K=80 for 300 units, AAPL call option with K=510 for 100 unit, PEP put option with K=87.5 for 200 units, and IBM put option with K=210 for 100 units.
2. We analyze the options that we bought on 2 April 2014. The prices of both call and put options of PEP are increasing so we sell PEP call and put options. On the other hand, the put option of AAPL is sold. The call option of AAPL decreases so we do not sell it.
3. The asset value on 7 April 2014 had risen from \$50000 to \$51474.

The detail of the transactions on 2 and 7 April is shown in Table 2. The process of evaluation for each trading day are made in the similar way. In the last day of trading, the return of our portfolio is calculated for about $return = \frac{54499-50000}{50000} = 0.08998$ or $8.998\,\%$.

5 Conclusions

Implementation of Fuzzy System in Binomial CRR model could help the investor to adjust their portfolio. Regarding to the above procedure, a better approximation to the price of the American option based on the Binomial CRR model has been given. By applying the fuzzy binomial CRR model, the richer information allows investors to adjust their portfolios. As a result, different trading strategies can be conducted to win as much possible. Using the simplified trading market, the portfolio gives return of $8.998\,\%$ only for 5 trading days.

For future studies, other variables in binomial option pricing model can also be fuzzified in order to get a better model.

Buy/Sell		Units	Price	Option Value				Asset value		gain/loss
Opsi			K	Call		Put				
				ask	bid	ask	bid			
Buy call PEP, K=80		400	80	3.25	2.15			1300	50000	
Buy call AAPL, K=470		200	470	71.9	71			14380	50000	
Buy put PEP, K=85		300	85			2.65	2.2	795	50000	
Buy put AAPL, K=570		200	570			33.36	31.2	6672	50000	
	call PEP K=80	400	80	2.94	4.05			1620	51474	1474
	call AAPL K=470	200	470	66.4	67.7			13540		
	put PEP K=85	300	85			2.94	2.87	861		
	put AAPL K=570	200	570			47.95	43	8600		
Buy call PEP, K=80		300	80	4.05	2.94			1215		
Buy call AAPL, K=510		100	510	21.63	18.9			2163		
Buy put PEP, K=87,5		200	87.5			3.85	3.5	770		
Buy put IBM, K=210		100	210			15.8	15.35	1580		
Sell call PEP, K=80		400	72.5	2.94	4.05			-1620		
Sell put PEP, K=85		300	85			2.94	2.87	-861		
Sell put AAPL, K=570		200	570			47.95	43	-8600	51474	1474

Table 2. Detail transaction on 2 and 7 April 2015

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