Application of Homotopy Analysis Method for Solving the SEIR Models of Epidemics

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Abstract
In this paper, an analytic technique, namely Homotopy Analysis Method (HAM) has been applied for solving SEIR model. The method (HAM) provides a direct scheme for solving the model, and also provides a convenient way of guaranteeing the convergence of solution series, so that it is valid for highly non-linear problems. In this respect, we
used the HAM which introduced a non-zero auxiliary parameter $h$ to construct a two-parameter family of equations (the zeroth-order deformation equation). The result of the theoretical analysis of the HAM shows that it yields a more accurate results in few iterations. We present and discuss graphical results to illustrate the solutions.

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**Keywords:** epidemic models, equilibrium states, stability, epidemiology

1 Introduction

The spread of infectious diseases has always been of concern and a threat to public health. It causes serious problem for the survival of human beings, other species, and for the economic and social development of human society. Dynamic models for infectious diseases are mostly based on compartment structures that were initially proposed by Kermack and McKendrick (1927) and developed later by many other biomathematicians. To formulate a model for the transmission of an epidemic disease, the population in a given region is often divided into several different groups or compartments whose sizes change with time.

In the compartment model studied by Kermack and McKendrick in 1927, the population is divided into three compartments; a susceptible compartment, labeled S, in which all the individuals are susceptible if they contact with a disease; an infected compartment, labeled I, in which all the individuals are infected by the disease and infectious; and a removed compartment, labeled R, in which all the individuals are removed or recovered from the infection. Denote the numbers of individuals in the compartments S, I, and R, at time $t$, as $S(t)$, $I(t)$, and $R(t)$, respectively.

When diseases like influenza, measles, rubella and chickenpox spread in a population rapidly, for a relatively short time, usually the vital dynamic functions, such as birth and death of the population, can be neglected in the models. A sequence of letters, such as SI describes the movement of individuals between classes: Susceptible become infectious. The infectives cannot be recovered from infection. To model diseases which confer permanent immunity and which are endemic because of births of new susceptible, SIR or SEIR models with vital dynamics are suitable. The vital dynamics are needed to avoid explosion of the population size. Epidemic models has been widely used in different forms for studying epidemiological processes such as the spread of influenza [12] and SARS [1,11,18] and even for the spread of rumors [19,20]. Epidemics models are also applied to modelling of STI epidemics, but not
all epidemic models are suitable for STIs since the sexual network plays an important role in the spread of disease [3].

The SEIR mathematical model in [21] based on compartmentalizing the population into susceptible, exposed, infected and infectious, and removed compactments was adopted. In this model, an exposed compartment refers to group of individuals that are infected but not yet infectious, is introduced. Let $E(t)$ denote the number of individuals in the exposed compartment at time $t$. We assume the population consist of four types of individuals, whose numbers are denoted by the letters $S,E,I$ and $R$ (which is why this is called SEIR model). All these are functions of time.

$S(t)$ is the number of susceptible, who do not have the disease but are vulnerable.

$E(t)$ is the number of exposed, who have the disease and cannot transmit it to others.

$I(t)$ is the number of infectives, who have the disease and can transmit it.

$R(t)$ is the number of removed, who cannot get the disease or transmit it: either they have a natural immunity, or they have recovered from the disease and immune from getting it again, or they have been placed in isolation, or they have died. Assume there is a steady contact rate between susceptible and infectives and that a constant proportion of these contact result in transmission. Then in time $\frac{\partial}{\partial t}$, $\partial S$ of the susceptible become exposed, where $\partial S = -\beta SI \partial t$ and $\beta$ is a positive constant. If $\omega > 0$ is the rate at which current exposed become infectious, then $\partial E = \beta SI \partial t - \omega Et$, if $\gamma > 0$ is the rate at which current infectious become isolated, then $\partial I = \omega Et - \gamma It$, and the number of new isolates $\partial R$ is given by $\partial R = \gamma It$. If we let $\partial t \rightarrow 0$, the following non linear systems of ODEs determines the progress of the disease.

\[
\begin{align*}
\frac{dS}{dt} &= -\beta SI \\
\frac{dE}{dt} &= \beta SI dt - \omega E dt \\
\frac{dI}{dt} &= \omega E dt - \gamma I dt \\
\frac{dR}{dt} &= \gamma I dt
\end{align*}
\]

with initial conditions

\[
S(0) = N_S, E(0) = N_E, I(0) = N_I, R(0) = N_R
\]

A lot of work has been done on the application of HAM in different aspect of mathematical modelling. In the works by [2,4,5,7,13,14,15,16,17], considerable
effort has been made in demonstrating the accuracy and convergence of HAM. In the referred works, it has been established that HAM converges faster and it is more accurate in solving non-linear ODE. In this paper, the HAM [9,10] is used to solve an SEIR epidemic model. An example is tested, and the obtained results suggest that HAM provides a useful analytic tool to investigate highly non-linear problems with multiple solutions and singularity in epidemic models.

2 Basic Idea of Homotopy Analysis Method

In this section, we apply the homotopy analysis method to the problem (1)-(4).

Consider the following equation:

\[ N[\vartheta(t)] = 0 \] (5)

where \( N \) is a nonlinear operator, \( t \) denotes the independent variable, \( \vartheta(t) \) is an unknown function. Let \( \vartheta_0(t) \) denote an initial guess of the exact solution \( \vartheta(t) \), \( h \neq 0 \) an auxiliary parameter, \( H(t) \neq 0 \) an auxiliary function, and \( L \) an auxiliary linear operator with the property \( L[\vartheta(t)] = 0 \) when \( \vartheta(t) = 0 \). Then using \( \rho \epsilon [0, 1] \) as an embedding parameter, we construct such a homotopy

\[
(1 - \rho)L[\phi(t; \rho) - \vartheta_0(t)] - \rho h H(t) N[\phi(t; \rho)] = \hat{H}[\phi(t; \rho); \vartheta_0(t), H(t), h, \rho] \] (6)

we have freedom to choose the initial guess \( \vartheta_0(t) \), the auxiliary linear operator \( L \), the non-zero auxiliary parameter \( h \), and the auxiliary function \( H(t) \).

Equating the homotopy (6) to zero, i.e.,

\[ \hat{H}[\phi(t; \rho); \vartheta_0(t), H(t), h, \rho] = 0 \]

we have the so-called zero order deformation equation

\[
(1 - \rho)L[\phi(t; \rho) - \vartheta_0(t)] = \rho h H(t) N[\phi(t; \rho)] \] (7)

when \( \rho = 0 \), the zero-order deformation equation (7) becomes

\[ \phi(t; 0) = \vartheta_0(t) \] (8)

and when \( \rho = 1 \), since \( h \neq 0 \) and \( H(t) \neq 0 \), the zero-order deformation equation (7) is equal to

\[ \phi(t; 1) = \vartheta(t) \] (9)

based on (8) and (9), as the embedding parameter \( \rho \) increases from 0 to 1, \( \phi(t; \rho) \) varies continuously from the initial approximation \( \vartheta_0(t) \) to the exact solution \( \vartheta(t) \).
Expanding $\phi(t; \rho)$ in Taylor series with respect to $\rho$, we have

$$
\phi(t; \rho) = \vartheta_0(t) + \sum_{m=1}^{\infty} \vartheta_m(t) \rho^m
$$

(10)

Where

$$
\vartheta_m(t) = \frac{1}{m!} \left[ \frac{\partial^m \phi(t; \rho)}{\partial \rho^m} \right]_{\rho=0}
$$

(11)

if the auxiliary linear operator, the initial quess, the auxiliary parameter $h$, and the auxiliary function are properly chosen so that

a. The solution $\phi(t; \rho)$ of the zero-order deformation equation (7) exists for all $\rho \in [0, 1]$

b. The deformation derivative $\left. \frac{\partial^m \phi(t; \rho)}{\partial \rho^m} \right|_{\rho=0}$ exist for $m = 1, 2, ...$

c. The power series (10) $\phi(t; \rho)$ converges at $\rho = 1$.

Then, we have under these assumptions the solution series.

$$
\phi(t; 1) = \vartheta_0(t) + \sum_{m=1}^{\infty} \vartheta_m(t) \rho^m
$$

(12)

According to equation (11), the governing equation can be deduced from the zero-order deformation equation (7). Define the vector

$$
\vec{\vartheta}_m = \{\vartheta_0(t), \vartheta_1(t), \vartheta_2(t), ..., \vartheta_n(t)\}
$$

(13)

Differentiating the zero-order deformation equation (7) in times with respect to $\rho$ and then dividing by $m!$ and finally setting $\rho = 0$, we have the so-called $m$th-order deformation equation.

$$
L \left[ \vartheta_m(t) - \chi_m \vartheta_{m-1}(t) \right] = h H(t) \Re_m(\vartheta_{m-1}(t))
$$

(14)

where

$$
\Re(\vartheta_{m-1}(t)) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(t; \rho)]}{\partial \rho^{m-1}}
$$

(15)

and

$$
\chi_m = \begin{cases} 
0, & m \leq 1 \\
1, & m > 1
\end{cases}
$$

(16)

It should be emphasized that $\vartheta_0(t)$ for $m \geq 1$ is governed by linear equation (11) with the linear boundary conditions that comes from the original problem, which can be solved easily by Maple.
3 Solution of the SEIR Model by Homotopy Analysis Method (HAM)

In order to explicitly construct approximate non-perturbative solutions of the system described by eqs (1)-(4), HAM well explained in [9,10] is employed. The advantage of this method is that it provides a direct scheme for solving the problem. To apply the HAM, we choose

\[ S_0(t) = N_S, E_0(t) = N_E, I_0(t) = N_I, R_0(t) = N_R \]

as initial approximation of \( S(t), E(t), I(t) \) and \( R(t) \). Let \( \rho \in [0,1] \) be the so-called embedding parameter. The HAM is based on the kind of continuous mappings.

\[ S(t) \rightarrow \phi_1(t; \rho), E(t) \rightarrow \phi_2(t; \rho), I(t) \rightarrow \phi_3(t; \rho), R(t) \rightarrow \phi_4(t; \rho) \]

such that, as the embedding parameter \( \rho \) increases from 0 to 1, \( \phi_i(t; \rho) \) varies from the initial approximation to the exact solution. To ensure this, choose such auxiliary linear operators as

\[ L_i[\phi_i(t; \rho)] = \frac{\partial \phi_i(t; \rho)}{\partial t}, i = 1, 2, 3, 4 \]

with the property

\[ L_i[C_i] = 0 \]

where \( C_i \) are integral constants. We define the non linear operators

\[ N_1[\phi_i(t; \rho)] = \frac{\partial \phi_i(t; \rho)}{\partial t} + \beta \phi_i(t; \rho)\phi_3(t; \rho), \]

\[ N_2[\phi_i(t; \rho)] = \frac{\partial \phi_i(t; \rho)}{\partial t} - \beta \phi_1(t; \rho)\phi_i(t; \rho) + \omega \phi_i(t; \rho), \]

\[ N_3[\phi_i(t; \rho)] = \frac{\partial \phi_i(t; \rho)}{\partial t} - \omega \phi_2(t; \rho) + \gamma \phi_i(t; \rho), \]

\[ N_4[\phi_i(t; \rho)] = \frac{\partial \phi_i(t; \rho)}{\partial t} - \gamma \phi_3(t; \rho), \]

let \( h_i \neq 0 \) and \( H_i(t) \neq 0 \) denote the so-called auxiliary parameter and auxiliary function, respectively. Using the embedding parameter \( \rho \), we construct a family of equations

\[ (1 - \rho)L[\phi_1(t; \rho) - S_0(t)] = \rho h_1 H_1(t)N_1[\phi_1(t; \rho)], \]

\[ (1 - \rho)L[\phi_2(t; \rho) - E_0(t)] = \rho h_2 H_2(t)N_2[\phi_2(t; \rho)], \]
Application of homotopy analysis method

\[(1 - \rho)L[\phi_3(t; \rho) - I_0(t)] = \rho h_3 H_3(t) N_3[\phi_3(t; \rho)],\]
\[(1 - \rho)L[\phi_4(t; \rho) - R_0(t)] = \rho h_4 H_4(t) N_4[\phi_4(t; \rho)],\]

subject to the initial conditions
\[\phi_1(0; \rho) = S_0, \phi_2(0; \rho) = E_0, \phi_3(0; \rho) = I_0, \phi_4(0; \rho) = R_0\]

By Taylor’s theorem, we expand \(\phi_i(t; \rho)\) by a power series of the embedding parameter \(\rho\) as follows

\[\phi_1(t; \rho) = S_0(t) + \sum_{m=1}^{\infty} S_m(t) \rho^m\]
\[\phi_2(t; \rho) = E_0(t) + \sum_{m=1}^{\infty} E_m(t) \rho^m\]
\[\phi_3(t; \rho) = I_0(t) + \sum_{m=1}^{\infty} I_m(t) \rho^m\]
\[\phi_4(t; \rho) = R_0(t) + \sum_{m=1}^{\infty} R_m(t) \rho^m\]

where
\[S_m(t) \frac{1}{m!} \frac{\partial^m \phi_1(t; \rho)}{\partial \rho^m} \bigg|_{\rho=0}\]
\[E_m(t) \frac{1}{m!} \frac{\partial^m \phi_2(t; \rho)}{\partial \rho^m} \bigg|_{\rho=0}\]
\[I_m(t) \frac{1}{m!} \frac{\partial^m \phi_3(t; \rho)}{\partial \rho^m} \bigg|_{\rho=0}\]
\[R_m(t) \frac{1}{m!} \frac{\partial^m \phi_4(t; \rho)}{\partial \rho^m} \bigg|_{\rho=0}\]

from the so-called \(m\)th-order deformation equation (14) and (15), we have

\[L[S_m(t) - \chi_m S_{m-1}(t)] = h_1 H_1(t) \Re_m(S_{m-1}(t))\]
\[L[E_m(t) - \chi_m E_{m-1}(t)] = h_2 H_2(t) \Re_m(E_{m-1}(t))\]
\[L[I_m(t) - \chi_m I_{m-1}(t)] = h_3 H_3(t) \Re_m(I_{m-1}(t))\]
\[L[R_m(t) - \chi_m R_{m-1}(t)] = h_4 H_4(t) \Re_m(R_{m-1}(t))\]

\[S_m(0) = 0, E_m(0) = 0, I_m(0) = 0, R_m(0) = 0\]
By the $h$–curves [9], it is reasonable to use $h_i = -1$. Using $H_i(t) = 1$, the $m$th-order deformation equation (16)-(19) for $m \geq 1$ becomes

$$S_m(t) = \chi_m S_{m-1}(t) - \int_0^t [S_{m-1}(\tau) + \beta \sum_{k=0}^{m-1} S_k(\tau) I_{m-1-k}(\tau)]d\tau \quad (20)$$

$$E_m(t) = \chi_m E_{m-1}(t) - \int_0^t [E_{m-1}(\tau) - \beta \sum_{k=0}^{m-1} S_k(\tau) I_{m-1-k}(\tau) + \omega E_{m-1}(\tau)]d\tau \quad (21)$$

$$I_m(t) = \chi_m I_{m-1}(t) - \int_0^t [I_{m-1}(\tau) - \omega E_{m-1}(\tau) + \gamma I_{m-1}(\tau)]d\tau \quad (22)$$

$$R_m(t) = \chi_m R_{m-1}(t) - \int_0^t [R_{m-1}(\tau) - \gamma I_{m-1}(\tau)]d\tau \quad (23)$$

4 Numerical Results and Discussion

The following values, for parameters, are considered for numerical results

<table>
<thead>
<tr>
<th>$N_S$</th>
<th>$N_E$</th>
<th>$N_I$</th>
<th>$N_R$</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>497</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.001</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

First to ninth terms approximations for $S(t)$, $E(t)$, $I(t)$ and $R(t)$ are calculated and presented below.

First terms approximations

$$S_1(t) = \sum_{m=0}^{1} S_m(t) = 20 - 2t$$

$$E_1(t) = \sum_{m=0}^{1} E_m(t) = 15 + 0.5t$$

$$I_1(t) = \sum_{m=0}^{1} I_m(t) = 10 + 1.3t$$

$$R_1(t) = \sum_{m=0}^{1} R_m(t) = 5 + 0.2t$$

Second terms approximation

$$S_2(t) = \sum_{m=0}^{2} S_m(t) = 20 - 2t - 0.03t^2$$
Application of homotopy analysis method

Fig.1. Plots of 20 terms approximation for $S(t)$, $E(t)$, $I(t)$ and $R(t)$ against time

$E_2(t) = \sum_{m=0}^{2} E_m(t) = 15 + 0.5t + 0.005t^2$

$I_2(t) = \sum_{m=0}^{2} I_m(t) = 10 + 1.3t - 0.012t^2$

$R_2(t) = \sum_{m=0}^{2} R_m(t) = 5 + 0.2t + 0.013t^2$

Third terms approximations:

$S_3(t) = \sum_{m=0}^{3} S_m(t) = 20 - 2t - 0.03t^2 + 0.0086666667t^3$

$E_3(t) = \sum_{m=0}^{3} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.00903333333t^3$
\[ I_3(t) = \sum_{m=0}^{3} I_m(t) = 10 + 1.3t - 0.012t^2 + 0.00008666666666t^3 \]

\[ R_3(t) = \sum_{m=0}^{3} R_m(t) = 5 + 0.2t + 0.013t^2 + 0.00008t^3 \]

Fourth terms approximations:

\[ S_4(t) = \sum_{m=0}^{4} S_m(t) = 20 - 2t - 0.03t^2 + 0.0088666667t^3 - 0.00006800000001t^4 \]

\[ E_4(t) = \sum_{m=0}^{4} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.00903333333t^3 + 0.00029433333t^4 \]

\[ I_4(t) = \sum_{m=0}^{4} I_m(t) = 10 + 1.3t - 0.012t^2 + 0.0000866666667t^3 - 0.0002262666666t^4 \]

\[ R_4(t) = \sum_{m=0}^{4} R_m(t) = 5 + 0.2t + 0.013t^2 + 0.00008t^3 - 0.00000043333333t^4 \]

Fifth terms approximations:

\[ S_5(t) = \sum_{m=0}^{5} S_m(t) = 20 - 2t - 0.03t^2 + 0.0088666667t^3 - 0.00006850000002t^4 \]

\[ -0.000011566t^5 \]

\[ E_5(t) = \sum_{m=0}^{5} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.00903333333t^3 + 0.000294333334t^4 \]

\[ + 0.00000567933337t^5 \]

\[ I_5(t) = \sum_{m=0}^{5} I_m(t) = 10 + 1.3t - 0.012t^2 + 0.0000866666667t^3 - 0.0002262666666t^4 \]

\[ + 0.00000679173332t^5 \]

\[ R_5(t) = \sum_{m=0}^{5} R_m(t) = 5 + 0.2t + 0.013t^2 + 0.00008t^3 - 0.00000043333332t^4 \]

\[ - 0.00000090506664t^5 \]
Six terms approximations:

\[ S_6(t) = \sum_{m=0}^{6} S_m(t) = 20 - 2t - 0.03t^2 + 0.0088666667t^3 - 0.0000685000002t^4 - 0.000011566t^5 - 0.000000812429997t^6 \]

\[ E_6(t) = \sum_{m=0}^{6} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.00903333333t^3 + 0.000294333332t^4 + 0.00000567933336t^5 + 0.00000007177744441t^6 \]

\[ I_6(t) = \sum_{m=0}^{6} I_m(t) = 10 + 1.3t - 0.012t^2 + 0.000086666667t^3 - 0.000226266666t^4 + 0.0000056791733334t^5 + 0.0000000720164452t^6 \]

\[ R_6(t) = \sum_{m=0}^{6} R_m(t) = 5 + 0.2t + 0.013t^2 + 0.00008t^3 - 0.000000433333332t^4 + 0.000000905066664t^5 + 0.0000000226391111t^6 \]

Seventh terms approximations:

\[ S_7(t) = \sum_{m=0}^{7} S_m(t) = 20 - 2t - 0.03t^2 + 0.0088666667t^3 - 0.0000685000002t^4 - 0.000011566t^5 - 0.000000812429997t^6 + 0.0000000408125619t^7 \]

\[ E_7(t) = \sum_{m=0}^{7} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.00903333333t^3 + 0.000294333332t^4 + 0.00000567933334t^5 + 0.00000007177744441t^6 - 0.000000005106648253t^7 \]

\[ I_7(t) = \sum_{m=0}^{7} I_m(t) = 10 + 1.3t - 0.012t^2 + 0.000086666667t^3 - 0.000226266666t^4 + 0.000006791733333t^5 + 0.0000000720164449t^6 + 0.0000000010048159361t^7 \]

\[ R_7(t) = \sum_{m=0}^{7} R_m(t) = 5 + 0.2t + 0.013t^2 + 0.00008t^3 - 0.000000433333332t^4 - 0.0000000905066664t^5 + 0.00000002263911112t^6 + 0.0000000020576127t^7 \]
Eighth terms approximations:

\[
S_8(t) = \sum_{m=0}^{8} S_m(t) = 20 - 2t - 0.03t^2 + 0.0088666667t^3 - 0.00006850000002t^4
- 0.000011566t^5 - 0.0000000812429997t^6 + 0.000000004081256189t^7
+ 0.00000003682268575t^8
\]

\[
E_8(t) = \sum_{m=0}^{8} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.0090333333333t^3 + 0.000294333334t^4
+ 0.0000567933337t^5 + 0.00000007177744439t^6 - 0.00000005106648252t^7
- 0.00000003043937543t^8
\]

\[
I_8(t) = \sum_{m=0}^{8} I_m(t) = 10 + 1.3t - 0.012t^2 + 0.0008666666667t^3 - 0.000226266666t^4
+ 0.00006791733332t^5 + 0.00000007201644457t^6 + 0.00000001004815936t^7
- 0.000000066345143t^8
\]

\[
R_8(t) = \sum_{m=0}^{8} R_m(t) = 5 + 0.2t + 0.013t^2 + 0.00008t^3 - 0.000000433333332t^4
- 0.0000000905066664t^5 + 0.0000000226391111t^6 + 0.000000020576127t^7
+ 0.0000000251203984t^8
\]

Ninth terms approximations:

\[
S_9(t) = \sum_{m=0}^{9} S_m(t) = 20 - 2t - 0.03t^2 + 0.0088666667t^3 - 0.0000685000000123t^4
- 0.00001156600001t^5 - 0.00000008124299842t^6 + 0.0000000408125615t^7
+ 0.00000004192425595t^8 - 0.00000007362733451t^9
\]

\[
E_9(t) = \sum_{m=0}^{9} E_m(t) = 15 + 0.5t + 0.005t^2 - 0.009033333401t^3 + 0.0002943333221t^4
+ 0.0000567933281t^5 + 0.00000007177744375t^6 - 0.00000005106648171t^7
- 0.00000003043937491t^8 + 0.00000001720687504t^9
\]
The Homotopy Analysis Method (HAM) yields rapidly convergent series solution by using a few iterations. The readers can consult (9,10) for further explanation on convergence of the Homotopy Analysis Method. Fig.1 shows plots of 20 terms approximations of S(t), E(t), I(t) and R(t). In Fig.1, we illustrate the case when small number of infectives $I_0$ into a susceptible population. An endemic situation occur and the number of infected individuals increases: The highest infective population (peak of epidemic) i.e $I_{max} = 154.7766$ occur when susceptible population has decreased to 28.5875. As time goes on i.e $t \to \infty$, eventually I approaches 0 and the disease is eradicated. the epidemic will end as $I \to 0$ with $S$ approaching some positive value $T_\infty=3.5320$. Where $S_{\infty}$ denotes the population who where never infected.

5 Discussion and Conclusion

It is well-known that nonlinear ordinary differential equations (ODEs) are difficult to solve than linear ODEs, especially by means of analytic methods. From our numerical example, we demonstrated the ability of HAM to converge very fast, we saw that the HAM converges in just nine (9) iterations. For the accuracy of the method, it has been established by [9,10]. we can then conclude that HAM is very efficient and accurate in solving SEIR model.

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