

On Continued Fraction of Order Twelve

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Abstract

In this paper, we establish some new modular relations between a continued fraction $V(q)$ of order 12 (established by M. S. Mahadeva Naika, B. N. Dharmendra and K. Shivashankara and recently studied this continued fraction by K. R. Vasuki, Abdulrawf A. A. Kathtan, G. Sharath, and C. Sathish Kumar) and $V(q^n)$ for $n=6,10,14,18$.

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1. INTRODUCTION

In Chapter 16 of his second notebook [1], [5], Ramanujan develops the theory of theta-function and is defined by

$$(1.1) \quad f(a, b) := \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}}, |ab| < 1,$$

$$= (-a; ab)_{\infty} (-b; ab)_{\infty} (ab; ab)_{\infty}$$

where $(a; q)_0 = 1$ and $(a; q)_{\infty} = (1 - a)(1 - aq)(1 - aq^2) \dots$

Following Ramanujan, we defined

$$(1.2) \quad \varphi(q) := f(q, q) = \sum_{n=-\infty}^{\infty} q^{n^2} = \frac{(-q; -q)_{\infty}}{(q; -q)_{\infty}},$$

$$(1.3) \quad \psi(q) := f(q, q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2; q^2)_{\infty}}{(q; q^2)_{\infty}},$$

$$(1.4) \quad f(-q) := f(-q, -q^2) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q; q)_{\infty}$$

and

$$(1.5) \quad \chi(q) := (-q; q^2)_{\infty}.$$

The ordinary hypergeometric series ${}_2F_1(a, b; c; x)$ is defined by

$${}_2F_1(a, b; c; x) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} x^n,$$

where

$$(a)_0 = 1, (a)_n = a(a+1)(a+2)\dots(a+n-1), \text{ for } n \geq 1, |x| < 1.$$

Let

$$z(r) := z(r; x) := {}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; x\right)$$

and

$$q_r := q_r(x) := \exp\left(-\pi \csc\left(\frac{\pi}{r}\right) \frac{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; 1-x\right)}{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; x\right)}\right).$$

where $r = 2, 3, 4, 6$ and $0 < x < 1$.

Let n denote a fixed natural number, and assume that

$$(1.6) \quad n \frac{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; 1-\alpha\right)}{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; \alpha\right)} = \frac{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; 1-\beta\right)}{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; \beta\right)},$$

where $r = 2, 3, 4$ or 6 . Then a modular equation of degree n in the theory of elliptic functions of signature r is a relation between α and β induced by (1.6). We often say that β is of degree n over α and $m(r) := \frac{z(r:\alpha)}{z(r:\beta)}$ is called the multiplier. We also use the notations $z_1 := z_1(r) = z(r : \alpha)$ and $z_n := z_n(r) = z(r : \beta)$ to indicate that β has degree n over α . When the context is clear, we omit the argument r in q_r , $z(r)$ and $m(r)$.

The class invariant G_n is defined by

$$(1.7) \quad G_n := 2^{-\frac{1}{4}} q^{-\frac{1}{24}} \chi(q) = (4\alpha(1-\alpha))^{-\frac{1}{24}}.$$

The celebrated Rogers-Ramanujan continued fraction is defined as

$$(1.8) \quad R(q) := \frac{q^{1/5} f(-q, -q^4)}{f(-q^2, -q^3)} = \frac{q^{1/5}}{1} + \frac{q}{1} + \frac{q^2}{1} + \frac{q^3}{1} + \dots, \quad |q| < 1,$$

On page 365 of his Lost Notebook [17], Ramanujan recorded five identities showing the relationships between $R(q)$ and five continued fractions $R(-q)$, $R(q^2)$, $R(q^3)$, $R(q^4)$, and $R(q^5)$. He also recorded these identities at the scattered places of his Notebooks [16]. L. J. Rogers [18] established the modular equations relating $R(q)$ and $R(q^n)$ for $n=2, 3, 5$, and 11. The last of these equations cannot be found in Ramanujan's works. Recently K. R. Vasuki and S. R. Swamy [23] found the modular equation relating $R(q)$ with $R(q^7)$.

The Ramanujan's cubic continued fraction $G(q)$ is defined as

$$(1.9) \quad G(q) := \frac{q^{1/3}f(-q, -q^5)}{f(-q^3, -q^3)} = \frac{q^{1/3}}{1} + \frac{q+q^2}{1} + \frac{q^2+q^4}{1} + \dots, \quad |q| < 1,$$

The continued fraction (1.9) was first introduced by Ramanujan in his second letter to G. H. Hardy [12]. He also recorded the continued fraction (1.9) on page 365 of his Lost Notebook [17] and claimed that there are many results for $G(q)$ similar the results obtained for the famous Rogers-Ramanujan continued fraction (1.8).

Motivated by Ramanujan's claim, H. H. Chan [8], N. D. Baruah [4], Vasuki and B. R. Srivatsa Kumar [21] established the modular relations between $G(q)$ and $G(q^n)$ for $n=2,3,5,7,11$ and 13.

The Ramanujan Gllinita-Gordon continued fraction [12, p. 44], [10], [17] is defined as follows:

$$(1.10) \quad H(q) := \frac{q^{1/2}f(-q^3, -q^5)}{f(-q, -q^7)} = \frac{q^{1/2}}{1} + \frac{q^2}{1+q^3} + \frac{q^4}{1+q^5} + \dots, \quad |q| < 1,$$

Chan and S. S. Hang [8], M. S. Mahadeva Naika, S. Chandan Kumar and M. Manjunatha [13] have established some new modular relations for Ramanujan-Göllnitz-Gordon continued fraction $H(q)$ with $H(q^n)$ for $n = 6, 10, 14$ and 16 and also established their explicit evaluations, Mahadeva Naika, B. N. Dharmendra and S. Chandan Kumar [14] have also established some new modular relations for Ramanujan-Göllnitz-Gordon continued fraction $H(q)$ with $H(q^n)$ for $n = 2, 3, 4, 5, 7, 8, 9, 11, 13, 15, 17, 19, 23, 25, 29$ and 55 and Vasuki and Sri-vatsa Kumar [21] established the modular relations between $H(q)$ and $H(q^n)$ for $n=3,4,5$ and 11. Recently, B. Cho, J. K. Koo, and Y. K. Park [9] extended the result cited above for the continued fraction (1.10) to all odd prime p by computing the affine models of modular curves $X(\Gamma)$ with $\Gamma = \Gamma_1(8) \cap \Gamma_0(16p)$.

Motivated by the cited works on the continued fractions (1.8)-(1.10), in this paper, we established the modular relation between continued fraction $V(q)$ and $V(q^n)$ for $n = 6, 10, 14, 18$.

$$(1.11) \quad V(q) := \frac{qf(-q, -q^{11})}{f(-q^5, -q^7)} = \frac{q(1-q)}{(1-q^3)+} \frac{q^3(1-q^2)(1-q^4)}{(1-q^3)(1+q^6)+ \dots}$$

The continued fraction (1.11) was first established by Mahadeva Naika, Dharmendra and K. Shivashankara [15] as a special case of a fascinating continued fraction identity recorded by Ramanujan in his Second Notebook [16, p. 74], they have also established a modular relation between the continued fraction $V(q)$ and $V(q^n)$ for $n=3, 5$ and recently Vasuki, Abdulrawf A. A. Kaththan, Sharath, and C. Sathish Kumar [24] $V(q)$ and $V(q^n)$ for $n=7,9,11,13$.

2. PRELIMINARY RESULTS

In this section, we collect the necessary results required to prove our main results.

Lemma 2.1. [24] If $x := V(q)$ and $y := V(q^2)$, then

$$(2.1) \quad x^2 - y + 2xy - x^2y + y^2 = 0.$$

Lemma 2.2. [24] If $x := V(q)$ and $y := V(q^3)$, then

$$(2.2) \quad x^3 - y^3 + y^2 - y + 3xy - 3x^2y^2 + x^3y^2 - x^3y = 0.$$

Lemma 2.3. [24] If $x := V(q)$ and $y := V(q^5)$, then

$$(2.3) \quad \begin{aligned} & xy^6 - 5x^6y^5 + 5x^5y^5 - 5x^4y^5 - 5xy^5 + 10x^3y^4 + 5xy^4 - 10x^4y^3 \\ & - 10x^2y^3 + 10x^3y^2 + 5x^5y^2 - 5x^5y - 5x^2y + 5xy - y + x^5 = 0. \end{aligned}$$

Lemma 2.4. [24] If $x := V(q)$ and $y := V(q^7)$, then

$$(2.4) \quad \begin{aligned} & 35x^4y^3 + y + 28x^2y^3 + 7x^3y^2 - 14xy^6 - xy^8 + x^8y^7 - 28x^5y^2 + 14x^5y - 35x^3y^4 \\ & + 21x^5y^5 + 35x^4y^5 - 7x^3y - 7xy - 28x^2y^2 + 14x^2y - x^7 + 7x^7y - 7x^3y^3 - 7x^6y \\ & + 28x^6y^2 - 14x^7y^2 + 7x^5y^3 - 7x^6y^3 + 7x^7y^3 - 35x^5y^4 - 7x^2y^5 + 7x^3y^5 - 7x^7y^7 \\ & - 7x^7y^5 + 28x^2y^6 - 28x^3y^6 + 7x^5y^6 - 28x^6y^6 + 7x^7y^6 + 7xy^7 - 7x^2y^7 + 7x^3y^7 \\ & - 7x^5y^7 + 14x^6y^7 + 7xy^2 - 7xy^3 = 0. \end{aligned}$$

Lemma 2.5. [24] If $x := V(q)$ and $y := V(q^9)$, then

$$\begin{aligned} & 153x^4y^3 + y + 297x^2y^3 + 171x^3y^2 + 99xy^6 + 9xy^8 - 45x^8y^7 - 99x^5y^2 + 9x^5y \\ & - 135xy^5 + 243x^5y^5 + 369x^4y^5 - 561x^6y^5 + 126xy^4 - 30x^3y - 9xy - 153x^2y^2 \\ & + 27x^2y + 10y^3 - 4y^2 - 16y^6 - 4y^8 + 18x^7y - 981x^3y^3 - 27x^6y + 165x^6y^2 \\ & - 135x^7y^2 + 198x^5y^3 - 333x^6y^3 + 243x^7y^3 - 369x^5y^4 + 369x^2y^5 + 378x^7y^5 \\ & - 243x^2y^6 + 333x^3y^6 - 153x^5y^6 + 432x^6y^6 - 297x^7y^6 - 54xy^7 + 135x^2y^7 \\ & - 165x^3y^7 + 54x^5y^7 - 171x^6y^7 + 153x^7y^7 + 45xy^2 - 90xy^3 - x^9y^8 - 9x^5y^8 \\ & + 30x^6y^8 - 99x^8y^3 - 18x^2y^8 + 4x^9y^7 + 9x^8y^8 - 9x^4y^8 - 10x^9y^6 + 90x^8y^6 \end{aligned}$$

(2.5)

$$\begin{aligned} & - 369x^7y^4 - 54x^4y^2 - 27x^7y^8 + 9x^4y + 135x^8y^4 + 27x^3y^8 + 558x^6y^4 - x^9 \\ & - 9x^8y + 54x^8y^2 - 10x^9y^2 - 3x^9y^4 + 4x^9y - 378x^2y^4 + 16x^9y^5 - 126x^8y^5 \\ & - 249x^4y^4 + 99x^4y^7 - 16y^4 + 19y^5 + 10y^7 + y^9 - 198x^4y^6 = 0. \end{aligned}$$

3. RELATION BETWEEN $H(q)$ AND $H(q^n)$

Theorem 3.1. If $u := H(q)$ and $v := H(q^6)$, then

(3.1)

$$\begin{aligned} & (-v^3 - 1 + v^2 + v^5)u^6 + (6v + 6v^4 - 6v^5 - 6v^2)u^5 + (9v^5 + 27v^3 \\ & - 27v^4 - 9v^2)u^4 + (-2v + 16v^4 - 2v^5 - 36v^3 + 16v^2)u^3 + (27v^3 \\ & - 27v^2 - 9v^4 + 9v)u^2 + (6v^2 - 6v + 6v^5 - 6v^4)u - v^3 + v^4 - v^6 + v = 0. \end{aligned}$$

Proof. Using the equations (2.1), then we have to express y in terms of x .

$$(3.2) \quad y = 1/2 - x + 1/2x^2 - 1/2\sqrt{1 - 4x + 2x^2 - 4x^3 + x^4}.$$

Replace q to q^2 in the equation (2.2) and using the above equation (3.2), we arrive at the equation (3.1). \square

Theorem 3.2. If $u := H(q)$ and $v := H(q^{10})$, then

$$\begin{aligned} & (-50v^{10} - 100v^8 + 5v^{11} + 5v^5 + 75v^7 - 30v^6 + 75v^9)u^{12} + (-50v^{11} - 10v^5 \\ & + 60v^6 - 550v^9 - 350v^7 + 600v^8 + 300v^{10})u^{11} + (-190v^4 + 195v^{11} + 580v^7 \\ & + 4v^6 - 1305v^8 - 1050v^{10} - 50v^2 - 1 + 125v^3 + 11v + 180v^5 + 1525v^9)u^{10} \\ & + (680v^4 - 790v^7 - 410v^{11} + 1840v^8 - 850v^5 - 280v^3 - 2550v^9 + 268v^6 \\ & + 40v^2 + 2100v^{10})u^9 + (230v^3 + 3385v^9 + 585v^{11} - 1020v^4 - 2950v^{10} \\ & + 1345v^7 + 60v^2 - 2120v^8 - 1134v^6 - 20v + 1675v^5)u^8 + (1120v^4 - 1900v^7 \\ & - 140v^3 + 40v + 3200v^{10} + 2000v^8 - 2300v^5 + 2240v^6 - 3460v^9 - 640v^{11} \\ & - 160v^2)u^7 + (2300v^5 + 2340v^7 - 3148v^6 - 1800v^8 + 520v^{11} - 840v^4 - 20v^3 \end{aligned}$$

(3.3)

$$\begin{aligned} & + 190v^2 - 2610v^{10} + 2820v^9 - 40v)u^6 + (3040v^6 - 320v^{11} + 20v^3 - 2300v^7 \\ & - 1660v^5 + 1440v^8 + 400v^4 - 40v^2 + 1640v^{10} - 1940v^9 + 8v)u^5 + (105v^3 \\ & - 740v^{10} - 2194v^6 + 140v^{11} - 160v^4 - 1060v^8 + 865v^5 + 1030v^9 - 130v^2 \\ & + 1675v^7 + 25v)u^4 + (220v^2 - 230v^5 - 190v^3 + 680v^8 + 240v^{10} + 1068v^6 \\ & - 40v^{11} - 850v^7 + 80v^4 - 50v - 480v^9)u^3 + (15v^{11} - 276v^6 - v^{12} + 35v \\ & - 190v^2 - 185v^4 - 190v^8 + 245v^3 + 176v^7 + 145v^9 + 80v^5 - 70v^{10})u^2 \\ & + (60v^6 - 110v^3 - 70v^5 - 10v - 2v^7 + 120v^4 + 60v^2)u - 6v^2 - 20v^4 \\ & + v^7 + 15v^3 - 10v^6 + v + 15v^5 = 0. \end{aligned}$$

Proof. Using the equations (3.2) and replace q to q^2 in the equation (2.3), we arrive at the equation (3.3). \square

Theorem 3.3. If $u := H(q)$ and $v := H(q^{14})$, then

(3.4)

$$\begin{aligned}
& (3528v^{13} + 252v^{14} - 182v^{15} + 3570v^{11} - 30212v^5 - 10682v^{12} + 1316v^{10} \\
& - 2086v^9 - 126v + 1750v^2 - 6650v^7 + 23226v^6 + 3164v^8 + 22526v^4 \\
& - 9058v^3)u^{11} + (24199v^9 - 14v - 20020v^4 - 14931v^{10} - 10626v^{11} \\
& - 63v^{15} + 20636v^{12} - 25368v^8 - 24878v^6 - 8155v^{13} + 7651v^3 - 1050v^2 \\
& + 28938v^5 + 16310v^7 + 1267v^{14})u^{10} + (77v^{15} - 1 - 1631v^6 + 385v^{13} \\
& + 798v^{11} - 147v^2 - 323v^8 + 1072v^7 + 1512v^5 - 959v^4 + 273v^9 + 469v^3 \\
& - 357v^{12} - 553v^{10} - 357v^{14} + 22v)u^{14} + (86v^8 - 560v^{11} - 196v^{13} + 84v^{14} \\
& + 476v^{10} + 378v^{12} - 252v^9 - 14v^{15} - 2v^7)u^{15} + (6104v^4 - 6146v^7 + 462v^8 \\
& + 868v^{14} - 2520v^3 - 56v - 3192v^{10} - 9688v^5 + 2408v^9 - 210v^{15} + 10136v^6 \\
& - 280v^{13} + 2156v^{11} + 588v^2 - 2310v^{12})u^{13} + (2177v^8 - 4067v^{11} - 84v^{13} \\
& - 20237v^6 - 1295v^2 + 294v^{15} + 23226v^5 - 1204v^{14} + 105v + 8260v^7 \\
& + 5712v^{10} + 5222v^{12} + 6433v^3 - 16359v^4 - 5943v^9)u^{12} + v + 28v^3 - 15v^2 \\
& - 56v^4 + 70v^5 - 49v^6 + 28v^7 + v^9 - 8v^8 + (53552v^8 + 8302v^4 + 29568v^{10} \\
& + 210v + 10584v^{13} - 2408v^{14} + 18970v^6 - 25774v^{12} + 16618v^{11} - 49378v^9 \\
& - 28014v^7 - 2366v^3 - 15288v^5 + 306v^{15} - 546v^2)u^9 + (62258v^9 + 7567v^4 \\
& - 11977v^6 - 21945v^{11} - 75331v^8 - 4277v^3 + 40922v^7 + 23492v^{12} + 2128v^2 \\
& + 2212v^{14} - 364v^{15} - 8505v^{13} - 364v - 38003v^{10} - 2317v^5)u^8 + (306v \\
& - 42658v^7 - 15960v^4 - 56910v^9 - 616v^{14} + 4200v^6 + 7700v^3 + 16940v^{11} \\
& + 1638v^{13} + 36050v^{10} - 10514v^{12} + 69722v^8 + 210v^{15} + 12614v^5 - 2562v^2)u^7 \\
& + (4683v^{13} - 4046v^{12} - 15918v^5 + 1512v^2 - 7112v^3 + 4312v^6 + 15862v^4 \\
& - 21v^{15} - 46480v^8 - 30352v^{10} + 30562v^7 - 4949v^{11} - 63v + 37618v^9 \\
& - 896v^{14})u^6 + (10066v^5 - 14042v^7 + 1652v^{14} + 4032v^3 + 18802v^8 - 182v \\
& - 5236v^{11} + 21210v^{10} + 12334v^{12} - 9184v^4 + 14v^2 - 7532v^6 - 7658v^{13} \\
& - 15554v^9 - 98v^{15})u^5 + (-1106v^2 - 1036v^3 - 15953v^{10} + 10759v^6 - 5663v^5 \\
& + 77v^{15} - 1148v^{14} - 12537v^{12} + 294v + 12978v^{11} + 6468v^9 - 1435v^7 - 679v^8 \\
& + 5789v^{13} + 3472v^4)u^4 + (112v^3 - 2072v^{13} - 28v^{15} - 210v + 2212v^7 \\
& - 7560v^{11} + 2436v^5 + 5320v^{12} - 1120v^4 - 616v^8 - 5782v^6 + 392v^{14} - 3850v^9 \\
& + 7980v^{10} + 1106v^2)u^3 + (-805v^{12} - 1337v^{10} + 336v^3 - v^{16} - 623v^4 + 224v^7 \\
& - 98v^{14} + 77v + 736v^9 - 43v^8 - 588v^2 + 1225v^{11} + 742v^5 + 357v^{13} + 63v^6 \\
& + 15v^{15})u^2 + (44v^8 + 154v^2 - 14v + 420v^4 - 252v^7 + 406v^6 - 2v^9 - 196v^3 \\
& - 560v^5)u + (-49v^{12} + 28v^9 + 70v^{11} + v^{15} - 8v^{14} + v^7 + 28v^{13} \\
& - 56v^{10} - 15v^8)u^{16} = 0.
\end{aligned}$$

Proof. Using the equations (2.1) and (2.4), we arrive at the equation (3.4). \square

Theorem 3.4. *If $u := H(q)$ and $v := H(q^{18})$, then*

$$\begin{aligned}
& (2v^{15} - 3v - 3193v^5 + 11v^{14} - 2v^2 + v^{17} + 515v^{13} - 7859v^6 + 4040v^7 + 6856v^{11} \\
& + 1640v^4 - 1 + 3v^{16} + 8061v^8 + 4982v^9 - 12237v^{10} - 592v^3 - 2782v^{12})u^{18} \\
& + (-81v + 209715v^9 + 135v^{17} + 245061v^7 - 945v^2 - 1917v^{14} + 1269v^{15} + 88140v^4 \\
& - 30429v^3 - 378v^{16} + 361698v^{11} - 495132v^6 - 152850v^5 - 645072v^{10} + 29880v^{13} \\
& - 36v^{15} - 142467v^{12} + 504603v^8)u^{16} + (-18912v^4 - 62592v^9 + 54v - 6138v^{13} \\
& + 149478v^{10} + 6660v^3 - 85524v^{11} - 126v^{14} + 33756v^{12} - 49788v^7 + 35670v^5 \\
& - 91404v^8 + 95658v^6 - 18v^{17})u^{17} + v + 2v^3 + 3v^2 + 576v^4 - 1736v^5 + 2985v^6 \\
& - 1453v^7 + 10131v^9 - 3448v^8 + 11965v^{11} - 13542v^{10} - 7064v^{12} - 11v^{15} - v^{18} \\
& - 531v^{14} + 2686v^{13} - 2v^{16} - 3v^{17} + (1115262v^{10} + 263184v^5 - 509856v^{11} \\
& + 1529898v^6 + 117642v^9 - 208950v^4 - 67734v^{13} - 546v^{17} + 19626v^{14} - 610572v^7 \\
& + 196224v^{12} + 5412v^2 - 1874736v^8 - 156v + 2634v^{16} - 9774v^{15} + 78750v^3)u^{15} \\
& + (41886v^{13} - 490431v^{10} + 1287v^{17} - 7056v^{16} - 3304776v^6 - 12681v^2 \\
& + 4649202v^8 + 27018v^{15} - 280941v^{11} + 1012689v^7 - 1856886v^9 + 185298v^{12} \\
& + 666v - 91761v^5 - 151074v^3 + 324528v^4 - 53352v^{14})u^{14} + (-7651260v^8 \\
& - 1188v - 445320v^4 - 1130346v^7 + 18900v^2 - 30312v^{15} + 61524v^{14} - 635652v^{12} \\
& + 4941918v^6 - 230688v^{10} - 83718v^5 - 1764v^{17} + 3801402v^9 + 7884v^{16} \\
& + 1088406v^{11} + 41202v^{13} + 231912v^3)u^{13} + (-4297173v^9 - 86085v^{14} \\
& + 513921v^4 + 17001v^{15} - 631776v^{11} + 778407v^7 + 21048v^{13} - 280170v^3 \\
& - 5395440v^6 + 1053v + 1170v^{17} + 449421v^{12} + 9092304v^8 - 5844v^5 - 18180v^2 \\
& - 675v^{16} - 197688v^{10})u^{12} + (-154386v^5 + 4352922v^9 - 621972v^{12} + 5097744v^6 \\
& - 13428v^{16} + 867618v^{11} - 414324v^4 + 100548v^{13} - 8410536v^8 + 49104v^{14} \\
& - 785502v^7 + 5382v^2 + 234v - 423846v^{10} + 468v^{17} + 20574v^{15} + 281070v^3)u^{11} \\
& + (-1935v - 237915v^3 - 2259v^{17} - 451794v^{11} + 242727v^5 - 4038567v^6 + 9045v^{14} \\
& + 252687v^4 - 2440479v^9 + 1234206v^7 + 146028v^{10} - 195174v^{13} + 13248v^2 \\
& + 26145v^{16} + 499557v^{12} - 60201v^{15} + 5109561v^8)u^{10} + (-1208566v^7 + 2914v \\
& + 1148536v^9 - 2078766v^8 - 77160v^4 - 854566v^{12} + 168072v^3 + 2914v^{17} \\
& - 79792v^{14} + 931322v^{11} - 737334v^{10} + 73644v^{15} - 26720v^2 + 2716562v^6 \\
& - 26720v^{16} + 404866v^{13} - 356100v^5)u^9 + (993363v^7 + 1563450v^{10} \\
& - 1597389v^{11} + 124866v^{14} + 26145v^2 + 1213467v^{12} - 93141v^3 - 2259v
\end{aligned}$$

$$\begin{aligned}
(3.5) \quad & + 13248v^{16} - 945330v^9 - 557553v^{13} - 42648v^4 + 651099v^8 - 1935v^{17} \\
& + 500763v^5 - 53451v^{15} - 1696971v^6)u^8 + (5382v^{16} + 27162v^3 + 66138v^7 \\
& + 468v + 66618v^4 - 2027634v^{10} - 108288v^{14} - 13428v^2 + 10962v^{15} + 234v^{17} \\
& + 1906314v^{11} + 627834v^6 - 1309830v^{12} + 1410180v^9 - 348096v^5 + 544524v^{13} \\
& - 905514v^8)u^7 + (-18180v^{16} + 25074v^{15} - 1075149v^7 - 695826v^6 + 339204v^5 \\
& + 1165959v^{12} - 403119v^{13} - 88239v^4 + 1870425v^8 + 1053v^{17} - 1974363v^{11} \\
& - 675v^2 + 16452v^3 + 1170v + 2186700v^{10} - 1769166v^9 + 57690v^{14})u^6 + (20124v^4 \\
& - 288030v^6 - 387642v^{12} - 30312v^3 + 48432v^5 - 1158114v^8 + 14850v^{14} + 7884v^2 \\
& + 96150v^{13} + 18900v^{16} - 1188v^{17} + 719400v^7 - 1195590v^{10} + 787728v^{11} \\
& - 1764v + 1363188v^9 - 38196v^{15})u^5 + (239229v^6 - 31113v^4 - 7056v^2 + 49788v^{13} \\
& + 775845v^{10} + 666v^{17} + 27018v^3 + 1287v - 578952v^7 + 878760v^8 + 103734v^{12} \\
& - 977373v^9 - 421380v^{11} + 33390v^{15} - 12681v^{16} - 53586v^{14} - 21096v^5)u^4 \\
& + (32628v^{14} + 2634v^2 - 47292v^5 + 334536v^{11} + 267168v^7 - 9774v^3 + 617328v^9 \\
& - 546v - 34164v^{13} - 15678v^{15} - 156v^{17} - 51930v^6 - 532482v^{10} - 484734v^8 \\
& + 22968v^4 - 92754v^{12} + 5412v^{16})u^3 + (135v + 20880v^{11} - 72198v^7 + 77310v^8 \\
& + 1269v^3 - 378v^2 + 39339v^6 - 50877v^9 - 6804v^5 + 1890v^4 - 945v^{16} + 6435v^{10} \\
& - 81v^{17} + 26685v^{13} - 10179v^{14} + 2511v^{15} - 33678v^{12})u^2 + (3132v^{14} \\
& - 67908v^{11} - 36v^3 + 54v^{17} - 18v - 20652v^6 - 53922v^9 + 72v^{15} + 13524v^7 \\
& - 3708v^4 + 40476v^{12} + 11472v^5 - 15456v^{13} + 75468v^{10} + 14196v^8)u = 0.
\end{aligned}$$

Proof. Using the equations (2.1) and (2.5), we arrive at the equation (3.5). \square

REFERENCES

- [1] C. Adiga, B. C. Berndt, S. Bhargava and G. N. Watson, Chapter 16 of Ramanujan's second notebook: Theta-function and q-series, Mem. Amer. Math. Soc., 53, No.315, Amer. Math. Soc., Providence, (1985).
- [2] Berndt, B.C.: Ramanujan's Notebooks, Part III. New York. Springer-Verlag 1991.
- [3] N. D. Baruah, Modular equations for Ramanujan's cubic continued fractions, 268, 244-255 (2002).
- [4] N. D. Baruah and R. Barman, Certain Theta-function identites and Ramanujan's modular equations of degree 3, Indian J. Math, 48, No. 1, 113-133 (2002).
- [5] B. C. Berndt, Ramanujan's Notebooks, Part III, Springer-Verlag, New York (1991).
- [6] B. C. Berndt, Ramanujan's Notebooks, Part V, Springer-Verlag, New York (1998).
- [7] H.H. Chan, On Ramanujan cubic continued fraction, Acta Arithm., 73, No, 343-355 (1995).
- [8] H.H. Chan, S.-S. Haung, On the Ramanujan-Göllnitz-Gordon continued fraction. Ramanujan J. 1, 75-90 (1997).
- [9] B. Cho, J. K. Koo, and Y. K. Park, Arithmetic of the Ramanujan-Göllnitz-Gordon continued fraction, J. Number Theory, 4, No. 129, 922-947 (2009).

- [10] H. Göllnitz, Partition mit Differenzbedingungen, *J. Reine Angew. Math.*, 25, 154-190 (1967).
- [11] B. Gordon, Some continued fractions of the Rogers-Ramanujan type, *Duke Math. J.*, 32, 741-748 (1965).
- [12] G. H. Hardy, Ramanujan, Chelsea, New York (1978).
- [13] M. S. Mahadeva Naika, S. Chandan Kumar and M. Manjunatha, On some new modular equations and their applications to continued fractions, (communicated).
- [14] M. S. Mahadeva Naika and B. N. Dharmendra and S. Chandankumar, Some identities for Ramanujan Göllnitz-Gordon continued fraction, *Aust. J. Math. Anal. Appl.* (to appear).
- [15] M. S. Mahadeva Naika, B. N. Dharmendra and K. Shivashankara, A continued fraction of order twelve. *Cent. Eur. J. Math.* 6(3), 393-404 (2008).
- [16] S. Ramanujan, Notebooks (2 volumes). Bombay. Tata Institute of Fundamental Research (1957).
- [17] S. Ramanujan, The ‘lost’ notebook and other unpublished papers. New Delhi. Narosa (1988).
- [18] L. J. Rogers, On a type of modular relation. *Proc. Lond. Math. Soc.*, 19, 387–397 (1921).
- [19] K. R. Vasuki and P. S. Guruprasad, On certain new modular relations for the Rogers-Ramanujan type functions of order twelve, *Proc. Jangjeon Math. Soc.* (to appear).
- [20] K. R. Vasuki, G. Sharath and K. R. Rajanna, Two modular equations for squares of the cubic functions with applications, *Note Math.* (to appear).
- [21] K. R. Vasuki and B. R. Srivatsa Kumar, Certain identities for Ramanujan-Göllnitz-Gordon continued fraction. *J. Compu. App. Math.* 187, 87-95 (2006).
- [22] K. R. Vasuki and B. R. Srivatsa Kumar, Two Identities for Ramanujan’s Cubic Continued Fraction, Preprint.
- [23] K. R. Vasuki and S. R. Swamy, A new identity for the Rogers-Ramanujan continued fraction. *J. Appl. Math. Anal. Appl.*, 2, No. 1, 71-83 (2006).
- [24] K. R. Vasuki, Abdulrawf A. A. Kathtan, Sharath, and C. Sathish Kumar, On a continued fraction of order 12. *Ukrainian Mathematical Journal*, Vol. 62, No. 12 (2011).

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