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An Approach for Solving Fuzzy Transportation Problem

Hadi Basirzadeh

Department of Mathematics
Shahid Chamran University
Ahvaz, Iran
basirzad@scu.ac.ir

Abstract

In this paper we shall study fuzzy transportation problem, and we introduce an approach for solving a wide range of such problem by using a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal or any LR fuzzy number. Thus, some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, in to crisp quantities by using our method and then by using the classical algorithms we solve and obtain the solution of the problem. The new method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function.

At the end, this method is illustrated with a numerical example.

Mathematics Subject Classification: 90C08, 90C90

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1 Introduction

The transportation problem is a special linear programming problem which arises in many practical applications. In this problem we determine optimal shipping patterns between origins or sources and destinations. Many problems which have nothing to do with transportation have this structure. Suppose that m origins are to supply n destinations with a certain product. Let a_i be the amount of the product available at origin i , and b_j be the amount of the product required at destination j. Further, we assume that the cost of shipping a unit amount of the product from origin i to destination j is c_{ij}, we then let x_{ij} represent the amount shipped from origin i to destination j. If shipping cost, are assumed to be proportional to the amount shipped from each origin to each destination so as to minimize total shipping cost turns out to be a linear programming problem. Transportation models have wide applications in logistics and supply chain for reducing the cost. When the cost coefficients and the supply and demand quantities are known exactly, many algorithms have been developed for solving the transportation problem. But, in the real world, there are many cases that the cost coefficients, and the supply and demand quantities are fuzzy quantities . A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. Shiang-Tai Liu and Chiang Kao [14], Chanas et al. [13], Chanas and Kuchta [12], proposed a method for solving fuzzy transportation problems. Nagoor Gani and Abdul Rezak [1] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Pandian et al. [9], proposed a method namely, fuzzy zero point methods, for finding a fuzzy optimal solution for a fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

In many fuzzy decision problems, the deta are represented in terms of fuzzy numbers. In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal. Thus, some fuzzy numbers are not directly comparable. Comparing between two or multi fuzzy numbers, and ranking such numbers is one of the important subjects, and how to set the rank of fuzzy numbers has been one of the main problems. Several method are introduced for ranking of fuzzy numbers. Here, we want to use a method which is introduced for ranking of fuzzy numbers, by Basirzadeh et al. [3]. Now, we want to apply this method for all fuzzy transportation problems, where all parameters can be trapezoidal fuzzy numbers, triangular fuzzy numbers, any LR fuzzy numbers, normal or abnormal fuzzy numbers. This method is very easy to understand and to apply. At the end, the optimal solution of a problem can be obtain in a fuzzy number or a crisp number form.

2 Mathematical formulation of a fuzzy transportation problem

Mathematically a transportation problem can be stated as follows:
minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij} \tag{1}$$

subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{m} x_{ij} = a_i, & i = 1, \cdots, m & \\ \sum_{i=1}^{m} x_{ij} = b_j, & j = 1, \cdots, m & \\ x_{ij} \geq 0 & i = 1, \cdots, m \quad , & j = 1, \cdots, m. \end{array}\right\} \tag{2}$$

where c_{ij} is the cost of transportation of an unit from the ith source to the jth destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the ith origin to jth destination. An obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \tag{3}$$

i.e. assume that total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has a feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum.

Mathematically a fuzzy transportation problem can be stated as follows:

minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{c_{ij}} x_{ij} \tag{4}$$

subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{m} x_{ij} = \tilde{a}_i, & i = 1, \cdots, m & \\ \sum_{i=1}^{m} x_{ij} = \tilde{b_j}, & j = 1, \cdots, m & \\ x_{ij} \geq 0 & i = 1, \cdots, m \quad , & j = 1, \cdots, m. \end{array}\right\} \tag{5}$$

in which the transportation costs $\tilde{c_{ij}}$, supply $\tilde{a}_i$ and demand $\tilde{b_j}$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem given in (4-5) to have a solution is that

$$\sum_{i=1}^{n} \tilde{a}_i \simeq \sum_{j=1}^{m} \tilde{b_j} \tag{6}$$

A considerable number of methods presented for fuzzy transportation problem. Some of them are based on ranking of the fuzzy numbers. Some of the methods for ranking of the fuzzy numbers for example, have limitations, are difficult in calculation, or they are non-intuitive, which makes them inefficient in practical applications, especially in the process of decision making. However, in some of these methods, as ones in which fuzzy numbers are compared according to their centeroid point (see [2], [15], [16]), the decision maker does not play any role in the comparison between fuzzy numbers. Nevertheless, there are certain methods in which fuzzy numbers are compared in a parametric manner (see e.g. [6], [5]).

Fuzzy and the nature of uncertainty is not always attributed to the inaccurate statistical information, but these conditions mainly occur in practice when we model linguistic expressions .

For this reason, when two fuzzy numbers are compared, it is quite natural that the result of the comparison would either be fuzzy or, at least, parametric, due to its subjective and interpretive nature. This can also be seen in the evolution of operators in the theory of fuzzy sets.

It is specifically seen in the theory of fuzzy decision that the parametric operators act better than non-parametric operators in case of experimental data [4]. Two factors play significant roles in fuzzy decision systems:

1. Contribution of the decision-maker in the decision making process,

2. Simplicity of calculation.

This paper attempts to propose a method for ranking and comparing fuzzy numbers to account for the above-mentioned factors as much as possible. The proposed method has also been compared with centroid point method (taking account of Wang's correction [16]).

3 Definition of an arbitrary fuzzy number

A fuzzy number has been defined in various forms. We appropriately employ the following definition of a fuzzy number [11]. We present an arbitrary fuzzy number $\tilde{A}_\omega$ by an ordered pair of functions $(\underline{A}(r), \overline{A}(r))$, where $0 \leq r \leq \omega$ and ω is an arbitrary constant between zero and one $(0 \leq \omega \leq 1)$, in a parametric form which satisfies the following requirements:

1. $\underline{A}(r)$ is a bounded left continuous non-decreasing function over $[0, \omega]$.
2. $\overline{A}(r)$ is a bounded left continuous non-increasing function over $[0, \omega]$.
3. $\underline{A}(r) \leq \overline{A}(r)$, $0 \leq r \leq \omega$.

A crisp number "k" is simply represented by $\underline{A}(r) = \overline{A}(r) = k$, $0 \leq r \leq \omega$. By appropriate definitions, the fuzzy numbers space $\{\underline{A}(r), \overline{A}(r)\}$ becomes a convex cone E^1 which is embedded isomorphic and isometric in a Banach space. If $\tilde{A}$ be an arbitrary fuzzy number then the α-cut of $\tilde{A}$ is $[\tilde{A}]_\alpha = [\underline{A}(\alpha), \overline{A}(\alpha)]$,$0 \leq \alpha \leq \omega$.
If $\omega = 1$, then the above-defined number is called a normal fuzzy number.

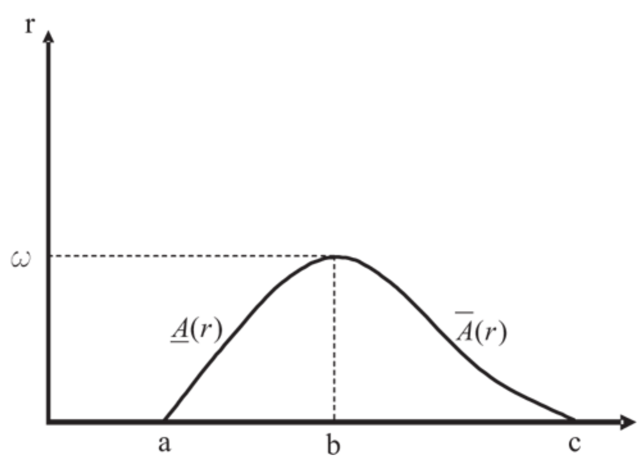

Figure 1: an arbitrary Fuzzy number $(0 \leq \omega \leq 1)$

Here $\tilde{A}_\omega$ represents a fuzzy number in which "ω" is the maximum membership value that a fuzzy number takes on. Whenever a normal fuzzy number is meant, the fuzzy number is shown by $\tilde{A}$, for convenience.

4 An approach for ranking fuzzy numbers

As mentioned earlier it seems that parametric methods of comparing fuzzy numbers, especially in fuzzy decision making theory, are more efficient than non-parametric methods. For example, in Cheng's centroid point method [2], fuzzy numbers are compared according to their Euclidean distances from the origin.

Negative fuzzy numbers in Cheng's centroid point method were not compared. However some time later Chu and Tsao [15] tried to solve this problem using the area between the centroid point to the origin. But their study was not flawless either. Abbasbandy and Assady [10] found that Chu and Tsao's area method occasionally causes non-intuitive ranking. They presented the sign distance method. But their method was non-parametric and only was applicable for normal fuzzy numbers.

It's clearly seen that non-parametric methods for comparing fuzzy numbers have some drawbacks in practice.

According to the above-mentioned definition of a fuzzy number, let $\tilde{A}_\omega = (\underline{A}(r), \overline{A}(r))$, $(0 \le r \le \omega)$ be a fuzzy number, then the value $M_\alpha(\tilde{A}_\omega)$, is assigned to $\tilde{A}_\omega$ for a decision level higher than "α" which is calculated as follows:

$$M_\alpha(\tilde{A}_\omega) = \frac{1}{2}\int_\alpha^\omega \{\underline{A}(r) + \overline{A}(r)\}dr, \quad where \quad 0 \le \alpha < 1$$

This quantity will be used as a basis for comparing fuzzy numbers in decision level higher than α.

Definition: A measure of a fuzzy number A is a function $M : F(X) \to R^+$ where $F(X)$ denotes the set of all fuzzy numbers on X. For each fuzzy number $\tilde{A}$, this function assigns a non-negative real number $M(\tilde{A})$ that expresses the measure of $\tilde{A}$.

The measure of a fuzzy number is obtained by the average of two side areas, left side area and right side area, from membership function to α axis.

The following requirements are essential:

(1). $M(A) = A$ iff A is a crisp number,

(2). $A \le B$ iff $M(A) \le M(B)$,

(3). If $\alpha \ge \omega$, then $M_\alpha(\tilde{A}_\omega) = 0$.

In order to clarify the concept of the above-mentioned quantity, consider the following fuzzy number:

$$\tilde{A}_\omega = (\underline{A}(r), \overline{A}(r)), \quad where \quad 0 \le r \le \omega$$

when a normal fuzzy number is meant, the fuzzy number is shown as follows:

$$\tilde{A} = (\underline{A}(r), \overline{A}(r)), \quad where \quad 0 \le r \le 1$$

It is clear that if $\alpha \ge \omega$, then $M_\alpha(\tilde{A}_\omega) = 0$. In order to clarify the concept of the above-mentioned quantity, consider the following fuzzy number:

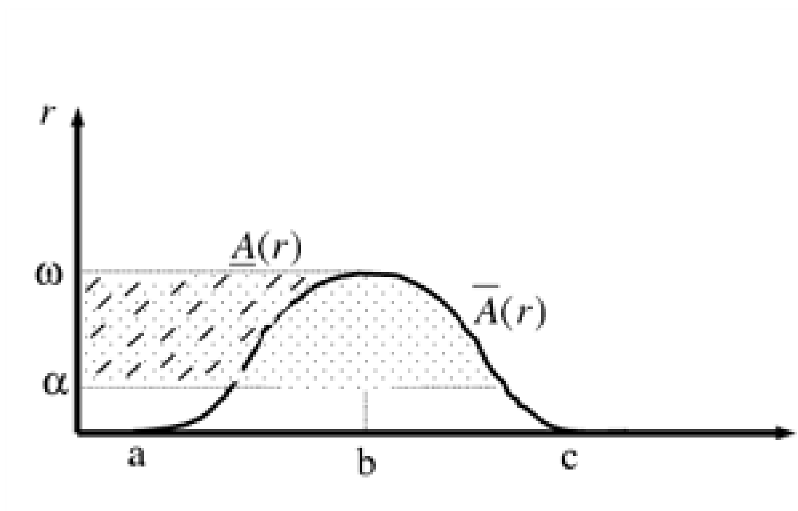

Figure 2: $M_\alpha(\tilde{A}_\omega)$ Quantity

As shown in fig.2, the presented quantity is the summation of the dotted area and the cross-hatched area.

$$M_\alpha(\tilde{A}_\omega) = \frac{1}{2}\int_\alpha^\omega \{\underline{A}(r) + \overline{A}(r)\}dr = \frac{1}{2}\int_\alpha^\omega \underline{A}(r)dr + \frac{1}{2}\int_\alpha^\omega \overline{A}(r)dr$$

Definition: If $\tilde{A}_\omega$ and $\tilde{B}_\omega$ are two arbitrary fuzzy numbers and $\omega, \omega' \in [0, 1]$, then we have:

1. $\tilde{A}_\omega \le \tilde{B}'_\omega \iff \forall\alpha \in [0, 1] \;\; M_\alpha(\tilde{A}_\omega) \le M_\alpha(\tilde{B}_{\omega'})$
2. $\tilde{A}_\omega = \tilde{B}_{\omega'} \iff \forall\alpha \in [0, 1] \;\; M_\alpha(\tilde{A}_\omega) = M_\alpha(\tilde{B}_{\omega'})$
3. $\tilde{A}_\omega \ge \tilde{B}_{\omega'} \iff \forall\alpha \in [0, 1] \;\; M_\alpha(\tilde{A}_\omega) \ge M_\alpha(\tilde{B}_{\omega'})$

Definition: If we compare two arbitrary fuzzy numbers including $\tilde{A}_\omega$ and $\tilde{B}_{\omega'}$ at decision levels higher than "α" and $\alpha, \omega, \omega' \in [0, 1]$, then we have:

1. $\tilde{A}_\omega \leq_\alpha \tilde{B}_{\omega'} \quad \Longleftrightarrow \quad M_\alpha(\tilde{A}_\omega) \leq M_\alpha(\tilde{B}_{\omega'})$
2. $\tilde{A}_\omega =_\alpha \tilde{B}_{\omega'} \quad \Longleftrightarrow \quad M_\alpha(\tilde{A}_\omega) = M_\alpha(\tilde{B}_{\omega'})$
3. $\tilde{A}_\omega \geq_\alpha \tilde{B}_{\omega'} \quad \Longleftrightarrow \quad M_\alpha(\tilde{A}_\omega) \geq_\alpha M_\alpha(\tilde{B}'_\omega)$

where $\tilde{A}_\omega \leq_\alpha \tilde{B}_{\omega'}$, i.e. at decision levels higher than α , $\tilde{B}_{\omega'}$ is greater than or equal to $\tilde{A}_\omega$.

If α is close to one, the pertaining decision is called a "high level decision ", in which case only parts of the two fuzzy numbers, with membership values between "α" and "1", will be compared. Likewise, if "α" is close to zero, the pertaining decision is referred to as a "low level decision", since members with membership values lower than both the fuzzy numbers are involved in the comparison. For instance, as shown in Fig.4, according to the presented quantity, the results clearly vary with different decision levels, e.g. $\tilde{A} \leq_{0.8} \tilde{B}$, $\tilde{A} \geq_{0.1} \tilde{B}$:

5 Triangular and Trapezoidal fuzzy numbers

Two relevant classes of fuzzy numbers, which are frequently used in practical purposes and are rather easy to work with, are "triangular and trapezoidal fuzzy numbers", as shown in Fig.5a and Fig.5b. some methods of approximating a fuzzy number with a trapezoidal fuzzy number, have also been presented (see e.g. [10], [8]) and hence there is no concern in this respect.

Definition: A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (δ, m, β) where δ,m and β are real number and its membership function $\mu_A(x)$ is given below,

$$\mu_A(x)= \begin{cases} 0 & for \quad x \leq \delta \\ (x-\delta)/(m-\delta) & for \quad \delta \leq x \leq m \\ 1 & for \quad x = m \\ (\beta - x)/(\beta - m) & for \quad m \leq x \leq \beta \\ 0 & for \quad x \geq \beta \end{cases}$$

According to the above-mentioned definition of a triangular fuzzy number, let $\tilde{A} = (\underline{A}(r), \overline{A}(r))$, $(0 \leq r \leq 1)$ be a fuzzy number, then the value $M(\tilde{A})$, is assigned to $\tilde{A}$ is calculated as follows:

$$M_0^{Tri}(\tilde{A}) = \frac{1}{2}\int_0^1 \{\underline{A}(r) + \overline{A}(r)\}dr = \frac{1}{4}[2m + \delta + \beta]$$

.

which is very convenient for calculation.

If $\tilde{A}_\omega = (\underline{A}(r), \overline{A}(r)) = (\delta + \frac{m-\delta}{\omega}r, \beta + \frac{m-\beta}{\omega}r)$ be an arbitrary triangular fuzzy number at decision level higher than "α" and $\alpha, \omega \in [0, 1]$, then the value $M_\alpha^{Tri}(\tilde{A}_\omega)$, assigned to $\tilde{A}_\omega$ may be calculated as follows:
If $\omega > \alpha$, then

$$M_\alpha^{Tri}(\tilde{A}_\omega) = \frac{1}{2}\int_\alpha^\omega \{\underline{A}(r) + \overline{A}(r)\}dr = \frac{1}{2\omega}[2m(\omega^2 - \alpha^2) + (\delta + \beta)(\omega - \alpha)^2].$$

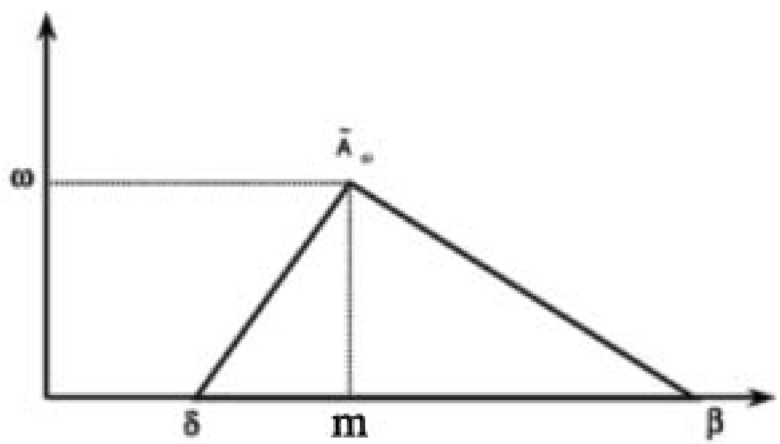

Figure 3: A triangular fuzzy number

Definition: A fuzzy number $\tilde{B}$ is a trapezoidal fuzzy number denoted by (δ, m, n, β) where δ,m,n and β are real number and its membership function $\mu_B(x)$ is given below,

$$\mu_B(x) = \begin{cases} 0 & for \quad x \leq \delta \\ (x-\delta)/(m-\delta) & for \quad \delta \leq x \leq m \\ 1 & for \quad m \leq x \leq n \\ (n-x)/(\beta-n) & for \quad n \leq x \leq \beta \\ 0 & for \quad x \geq \beta \end{cases}$$

According to the above-mentioned definition of a trapezoidal fuzzy number, let $\tilde{B} = (\underline{B}(r), \overline{B}(r))$, $(0 \leq r \leq 1)$ be a fuzzy number, then the value $M(\tilde{B})$, is assigned to $\tilde{B}$ is calculated as follows:

$$M_0^{Tra}(\tilde{B}) = \frac{1}{2}\int_0^1 \{\underline{B}(r) + \overline{B}(r)\}dr = \frac{1}{4}[m + n + \delta + \beta].$$

which is very convenient for calculation.
If $\tilde{B}_\omega = (\underline{B}(r), \overline{B}(r)) = (\delta + \frac{m-\delta}{\omega}r, \beta + \frac{n-\beta}{\omega}r)$ be an arbitrary trapezoidal fuzzy number at decision level higher than "α" and $\alpha, \omega \in [0,1]$, then the value $M_\alpha^{Tra}(\tilde{B}_\omega)$, assigned to $\tilde{B}_\omega$ may be calculated as follows:
If $\omega > \alpha$, then

$$M_\alpha^{Tra}(\tilde{B}_\omega) = \frac{1}{2}\int_\alpha^\omega \{\underline{B}(r) + \overline{B}(r)\}dr = \frac{1}{4}[m + n + \frac{\alpha}{\omega}(m + n - \delta - \beta) + \delta + \beta](\omega - \alpha).$$

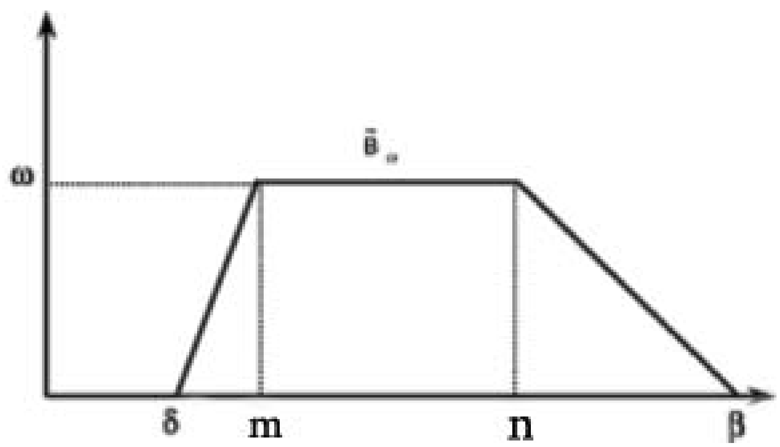

Figure 4: A trapezoidal fuzzy number

Obviously, if $\alpha \geq \omega$, then the above quantity will be zero. It can also be seen that if $\tilde{A}$ is a normal triangular or trapezoidal fuzzy number ($\omega = 1$) the above quantities reduce to:

$$M_\alpha^{Tri}(\tilde{A}) = \frac{1}{2}\int_\alpha^1 \{\underline{A}(r) + \overline{A}(r)\}dr = \frac{1}{2}[2m(1-\alpha^2) + (\delta + \beta)(1-\alpha)^2] \quad ,0 \leq \alpha < 1.$$

$$M_\alpha^{Tra}(\tilde{B}) = \frac{1}{2}\int_\alpha^1 \{\underline{B}(r) + \overline{B}(r)\}dr = \frac{1}{4}[m + n + \frac{\alpha}{1}(m + n - \delta - \beta) + \delta + \beta](1 - \alpha) \quad ,0 \leq \alpha < 1.$$

As the above relation show, if the fuzzy number is symmetrical ($\delta = \beta$), the relations may be simplified more (the second terms on the right-hand side of the above equations are canceled out). For simplicity, based on the results obtained, hereafter the triangular fuzzy number $\tilde{A}$ and the trapezoidal fuzzy number $\tilde{B}$ will be represented as $\tilde{A} = (\delta, m, \beta)$ and $\tilde{B} = (\delta, m, n, \beta)$ respectively.

The following definitions of the basic arithmetic operations on fuzzy numbers may be helpful [9].

Definition let $\tilde{A} = (\delta_1, m_1, \beta_1)$ and $\tilde{B} = (\delta_2, m_2, \beta_2)$ be two triangular fuzzy numbers. Then

(1). $\tilde{A} \oplus \tilde{B} = (\delta_1, m_1, \beta_1) \oplus (\delta_2, m_2, \beta_2) = (\delta_1 + \delta_2, m_1 + m_2, \beta_1 + \beta_2)$

(2). $\tilde{A} \ominus \tilde{B} = (\delta_1, m_1, \beta_1) \ominus (\delta_2, m_2, \beta_2) = (\delta_1 + \beta_2, m_1 - m_2, \beta_1 + \delta_2)$

(3). $\tilde{A} \otimes \tilde{B} = (\delta_1, m_1, \beta_1) \otimes (\delta_2, m_2, \beta_2) = (s_1, s_2, s_3)$

where

$s_1 = minimum\{\delta_1\delta_2, \delta_1\beta_2, \beta_1\delta_2, \beta_1\beta_2\}$,

$s_2 = mn$

$s_3 = maximum\{\delta_1\delta_2, \delta_1\beta_2, \beta_1\delta_2, \beta_1\beta_2\}$,

Definition let $\tilde{A} = (\delta_1, m_1, n_1, \beta_1)$ and $\tilde{B} = (\delta_2, m_2, n_2, \beta_2)$ be two trapezoidal fuzzy numbers. Then

(1). $\tilde{A} \oplus \tilde{B} = (\delta_1, m_1, n_1, \beta_1) \oplus (\delta_2, m_2, n_2, \beta_2) = (\delta_1 + \delta_2, m_1 + m_2, n_1 + n_2, \beta_1 + \beta_2)$

(2). $\tilde{A} \ominus \tilde{B} = (\delta_1, m_1, n_1, \beta_1) \ominus (\delta_2, m_2, n_2, \beta_2) = (\delta_1 - \beta_2, m_1 - n_2, n_1 - m_2, \beta_1 - \delta_2)$

(3). $\tilde{A} \otimes \tilde{B} = (\delta_1, m_1, n_1, \beta_1) \otimes (\delta_2, m_2, n_2, \beta_2) = (t_1, t_2, t_3, t_4)$

where

$t_1 = minimum\{\delta_1\delta_2, \delta_1\beta_2, \beta_1\delta_2, \beta_1\beta_2\}$,

$t_2 = minimum\{mn, m_1n_2, n_1m_2, n_1n_2\}$

$t_3 = maximum\{mn, m_1n_2, n_1m_2, n_1n_2\}$

$t_4 = maximum\{\delta_1\delta_2, \delta_1\beta_2, \beta_1\delta_2, \beta_1\beta_2\}$,

6 The new approach for solving fuzzy transportation problem

Now, we introduce a new method for solving a fuzzy transportation problem where the transportation cost, supply and demands are fuzzy numbers. The fuzzy numbers in each problem may be triangular or trapezoidal or any fuzzy numbers or mixture of them. The optimal solution for the fuzzy transportation problem can be obtain as a crisp or fuzzy form.

Step 1. Calculate the values M(.) for each fuzzy data, the transportation costs $\breve{c_{ij}}$, supply $\breve{a}_i$ and demand $\breve{b_j}$ quantities which are fuzzy quantities.

Step 2. By replacing $M(\breve{c_{ij}})$, $M(\breve{a}_i)$ and $M(\breve{b_j})$ which are crisp quantities instead of the $\breve{c_{ij}}$, $\breve{a}_i$ and $\breve{b_j}$ quantities which are fuzzy quantities, define a new crisp transportation problem.

Step 3. Solve the new crisp transportation problem, by usual method , and obtain the crisp optimal solution of the problem.

Note Any solution of this transportation problem will contain exactly (m+n-1) basic feasible solutions. We note that the optimal solution X_{ij} is to be some positive integer or zero, but the optimal solution X_{ij} for a crisp transportation problem may be integer or non integer, because the RHS of the problem are the measure of fuzzy numbers which are real numbers. If you accept the crisp solution then stop. The optimal solution is in your hand. If you want fuzzy form of solution, go to the next step.

Step 4. Determine the locations of non zero basic feasible solutions in transportation tableau. The basis is rooted spanning tree,that is there must be at least one basic cell in each row and in each column of the transportation tableau. Also, the basis, must be a tree, that is , the (m-n-1) basic cells should not contain a cycle. Therefore, there exist some rows and columns which have only one basic cell. By starting from these cells, calculate the fuzzy basic solutions, continue until obtain (m+n-1) basic solutions.

7 Numerical example

The following example may be helpful to clarify the proposed method:

Example : Consider the following fuzzy transportation problem which is in [9]. All of data in this problem are trapezoidal fuzzy numbers. We want to solve it by our method, and then we will compare the results.

	1	2	3	4	Supply
1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Table 1

According to the above-mentioned definition of a trapezoidal fuzzy number, let $\tilde{B} = (\underline{B}(r), \overline{B}(r))$, $(0 \leq r \leq 1)$ be a fuzzy number, then the value $M(\tilde{B})$, is assigned to $\tilde{B}$ is calculated as follows:

$$M_0^{Tra}(\tilde{B}) = \frac{1}{2}\int_0^1 \{\underline{B}(r) + \overline{B}(r)\}dr = \frac{1}{4}[m + n + \delta + \beta]$$

. Therefore, we obtain the values of $M(\tilde{A_{ij}})$,$M(\tilde{a_i})$ and $M(\tilde{b_j})$ according to the proposed method:

$\tilde{A_{11}} = (1, 2, 3, 4)$	$\mathrm{M}_0^{Tra}(\tilde{A_{11}}) = \frac{1}{4}(1 + 2 + 3 + 4) = 2.5$
$\tilde{A_{12}} = (1, 3, 4, 6)$	$\mathrm{M}_0^{Tra}(\tilde{A_{12}}) = \frac{1}{4}(1 + 3 + 4 + 6) = 3.5$
$\tilde{A_{13}} = (9, 11, 12, 14)$	$\mathrm{M}_0^{Tra}(\tilde{A_{13}}) = \frac{1}{4}(9 + 11 + 12 + 14) = 11.5$
$\tilde{A_{14}} = (5, 7, 8, 11)$	$\mathrm{M}_0^{Tra}(\tilde{A_{14}}) = \frac{1}{4}(5 + 7 + 8 + 11) = 7.75$
$\tilde{A_{21}} = (0, 1, 2, 4)$	$\mathrm{M}_0^{Tra}(\tilde{A_{21}}) = \frac{1}{4}(0 + 1 + 2 + 4) = 1.75$
$\tilde{A_{22}} = (-1, 0, 1, 2)$	$\mathrm{M}_0^{Tra}(\tilde{A_{22}}) = \frac{1}{4}(-1 + 0 + 1 + 2) = 0.5$
$\tilde{A_{23}} = (5, 6, 7, 8)$	$\mathrm{M}_0^{Tra}(\tilde{A_{23}}) = \frac{1}{4}(5 + 6 + 7 + 8) = 6.5$
$\tilde{A_{24}} = (0, 1, 2, 3)$	$\mathrm{M}_0^{Tra}(\tilde{A_{24}}) = \frac{1}{4}(0 + 1 + 2 + 3) = 1.5$
$\tilde{A_{31}} = (3, 5, 6, 8)$	$\mathrm{M}_0^{Tra}(\tilde{A_{31}}) = \frac{1}{4}(3 + 5 + 6 + 8) = 5.5$
$\tilde{A_{32}} = (5, 8, 9, 12)$	$\mathrm{M}_0^{Tra}(\tilde{A_{32}}) = \frac{1}{4}(5 + 8 + 9 + 12) = 8.5$
$\tilde{A_{33}} = (12, 15, 16, 19)$	$\mathrm{M}_0^{Tra}(\tilde{A_{33}}) = \frac{1}{4}(12 + 15 + 16 + 19) = 15.5$
$\tilde{A_{34}} = (7, 9, 10, 12)$	$\mathrm{M}_0^{Tra}(\tilde{A_{34}}) = \frac{1}{4}(7 + 9 + 10 + 12) = 9.5$

Table 2

and the fuzzy supplies are

$\tilde{a_1} = (1, 6, 7, 12)$	$\mathrm{M}_0^{Tra}(\tilde{a_1}) = \frac{1}{4}(1 + 6 + 7 + 12) = 6.5$
$\tilde{a_2} = (0, 1, 2, 3)$	$\mathrm{M}_0^{Tra}(\tilde{a_2}) = \frac{1}{4}(0 + 1 + 2 + 3) = 1.5$
$\tilde{a_3} = (5, 10, 12, 17)$	$\mathrm{M}_0^{Tra}(\tilde{a_3}) = \frac{1}{4}(5 + 10 + 12 + 17) = 11$

Table 3

and fuzzy demands are

$\tilde{b_1} = (5, 7, 8, 10)$	$\mathrm{M}_0^{Tra}(\tilde{b_1}) = \frac{1}{4}(5 + 7 + 8 + 10) = 7.5$
$\tilde{b_2} = (1, 5, 6, 10)$	$\mathrm{M}_0^{Tra}(\tilde{b_2}) = \frac{1}{4}(1 + 5 + 6 + 10) = 5.5$
$\tilde{b_3} = (1, 3, 4, 6)$	$\mathrm{M}_0^{Tra}(\tilde{b_3}) = \frac{1}{4}(1 + 3 + 4 + 6) = 3.5$
$\tilde{b_4} = (1, 2, 3, 4)$	$\mathrm{M}_0^{Tri}(\tilde{b_4}) = \frac{1}{4}(1 + 2 + 3 + 4) = 2.5$

Table 4

and the total fuzzy supply is $\tilde{S} = (6, 17, 21, 32)$ and the total fuzzy demand is $\tilde{D} = (8, 17, 21, 30)$, so

$\tilde{S} = (6, 17, 21, 32)$	$\mathrm{M}_0^{Tra}(\tilde{S}) = \frac{1}{4}(6 + 17 + 21 + 32) = 19$
$\tilde{D} = (8, 17, 21, 30)$	$\mathrm{M}_0^{Tra}(\tilde{D}) = \frac{1}{4}(8 + 17 + 21 + 30) = 19$

Table 5

Since $M_0^{Tra}(\tilde{S}) = M_0^{Tra}(\tilde{D})$, the given problem is a balanced one.
Now, by using our method we change the fuzzy transportation problem in to a crisp transportation problem. So, we have the following reduced fuzzy transportation problem:

	1	2	3	4	Supply
1	2.5	3.5	11.5	7.75	6.5
2	1.75	0.5	6.5	1.5	1.5
3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	

Table 6

As shown in the table 6, the result of defuzzification of the fuzzy numbers obtaining the measures which are not all integer. So, existing of a non integer value in a transportation problem follows this fact that the solution of the crisp transportation problem is not integer. We note that the solution in a usual

transportation problem is integer, because its matrix is an unimodular matrix [7]. If we solve the new problem, we obtain the solutions as follows:
$X_{12} = 5.5$, $X_{13} = 1$, $X_{23} = 1.5$, $X_{31} = 7.5$, $X_{33} = 1$, $X_{34} = 2.5$,

and the total crisp value of the problem is $X_0 = 121$.

	1	2	3	4	Supply
1		5.5	1		6.5
2			1.5		1.5
3	7.5		1	2.5	11
Demand	7.5	5.5	3.5	2.5	

Table 7

Now, we can return to initial problem and obtain the fuzzy solution of the fuzzy transportation problem based on the data of table-7.

	1	2	3	4	Supply
1		(1,5,6,10)	(-9,0,2,11)		(1,6,7,12)
2			(0,1,2,3)		(0,1,2,3)
3	(5,7,8,10)		(-9,-1,3,11)	(1,2,3,4)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Table 8

where the fuzzy optimal solution for the given fuzzy transportation problem is:
$\tilde{X_{12}} = (1, 5, 6, 10)$
$\tilde{X_{13}} = (-9, 0, 2, 11)$
$\tilde{X_{23}} = (0, 1, 2, 3)$
$\tilde{X_{31}} = (5, 7, 8, 10)$

$\tilde{X_{33}} = (-9, -1, 3, 11)$
$\tilde{X_{34}} = (1, 2, 3, 4)$

The results are the same as the results which obtained by Pandian et al. [9]. We note that the value of the variables are the same, but the value of the objective functions are different. The crisp value of the optimum fuzzy transportation cost for the given problem by Pandian method, is 132.17 where it obtained by our method 121. What seems to be clear is that there exists no uniquely method for comparing fuzzy numbers, and different methods may satisfy different desirable criteria.

8 Conclusion

In this paper, a simple yet effective parametric method was introduced to solve fuzzy transportation problem by using ranking of fuzzy numbers. This method can be used for all kinds of fuzzy transportation problem, whether triangular and trapezoidal fuzzy numbers with normal or abnormal data. The new method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function.

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