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Dominator Coloring of Prism Graph

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Abstract

A graph has a dominator coloring if it has a proper coloring in which each vertex of the graph dominates every vertex of some color class. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of *G*. In this paper the dominator chromatic number for Prism graph is studied and also the relation between dominator chromatic number, domination number and chromatic number is shown.

Keywords: Coloring, Domination, Dominator Coloring

1 Introduction

Let G= (V, E) be a graph such that V is the vertex set and E is the edge set. A dominating set S is a subset of the vertex set V of graph G such that every vertex in the graph either belongs to S or adjacent to S. The domination number γ (G) is the minimum cardinality of a dominating set of G.

A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices have different colors. A subset of vertices colored with the same color is called a color class. The chromatic number is the minimum number of colors needed in a proper coloring of a graph and is denoted by χ (G).

A dominator coloring of a graph G is a proper coloring of graph such that every vertex of V dominates all vertices of at least one color class (possibly its own class). i.e., it is coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other class and this concept was introduced by Ralucca Michelle Gera in 2006 [2]. The dominator chromatic number χ_d (G) is the minimum number of color classes in a dominator coloring of G. The relation between dominator chromatic number, chromatic number and domination number of some classes of graphs were studied in [1], [3]. The dominator coloring of bipartite graph, star and double star graphs, central and middle graphs, fan, double fan, helm graphs etc. were also studied in various papers [4], [6], [7], [8].

In this paper the dominator chromatic number for Prism graph is studied and also the relation between dominator chromatic number, domination number and chromatic number is shown.

A graph corresponding to the skeleton of an n-prism is called a prism graph and is denoted by Y_n. A prism graph also called as circular ladder graph, has $2n$ vertices and $3n$ edges. Prism graphs are both planar and polyhedral. It is equivalent to the generalized Petersen graph $P_{n,1}$ and the Cayley graph of the dihedral group D_{2n}.

2 Dominator Coloring of Prism graph

Theorem 2.1: For a prism graph Y_n, $n \geq 9$, $\chi_d(Y_n) = n + 1$

Proof:

The vertex set of the prism graph is given by $V = \{v_i | 1 \leq i \leq 2n\}$ and the edge set is defined as $E = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{ v_i v_{i+1} ; 1 \leq i \leq 2n-1\}$, $E_2 = v_n v_1$, $E_3 = v_{2n} v_{n+1}$ and $E_4 = \{ v_i v_{n+i+1} ; 1 \leq i \leq n-1\}$.

The following procedure gives a dominator coloring of prism graph Y_n.

Let the adjacent vertices of v_1, v_2 namely $\{v_2, v_n, v_{n+2}\}$, $\{v_1, v_3, v_{n+3}\}$ be assigned color 1 and the color 2 respectively. The vertex v_4 is given color 3. Let the vertices v_i, v_{n+i} for $5 \leq i \leq n-1$ be given color $i-1$. The vertices v_{2n}, v_{n+1}, v_{n+4} are given colors $n-1$, n, $n+1$ respectively.

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Then the vertices v_i, $1 \leq i \leq n$, dominate color class i. The vertices v_{n+1}, v_{n+2} dominate color class n and v_{n+3}, v_{n+4}, v_{n+5} dominate color class $n+1$. The vertex v_{n+i} for $6 \leq i \leq n$, dominates color class i-2.

Thus the minimum number of colors needed for dominator coloring of a prism graph Y_n for $n \geq 9$ is $n+1$. i.e., the dominator chromatic number is $\chi_d(Y_n) = n+1$, for $n \geq 9$.

Illustration 2.1: Figure 1 shows dominator coloring of prism graph Y_{14}.

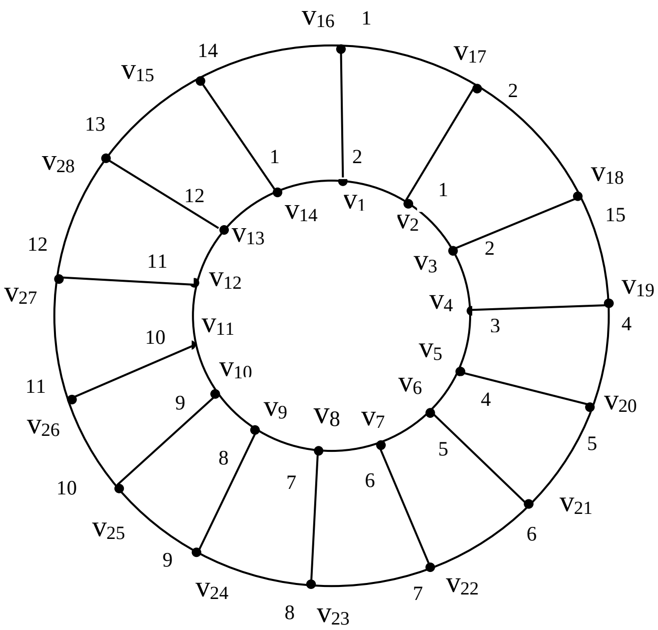

Figure 1

The vertices v_i, $1 \leq i \leq 14$, dominate color class i. The vertices v_{15}, v_{16} dominate color class 14 and v_{17}, v_{18}, v_{19} dominate color class 15. For $6 \leq i \leq 14$, the vertex v_{14+i} dominates color class i $-$ 2. Thus $\chi_d(Y_{14})$ = 15.

Observation 2.2: The dominator chromatic number of a prism graph Y_n, $3 \leq n \leq 8$, $n \neq 5$, is n i.e., $\chi_d(Y_n) = n$

Illustration 2.2: Figure 2 shows the dominator coloring of prism graph Y_6.

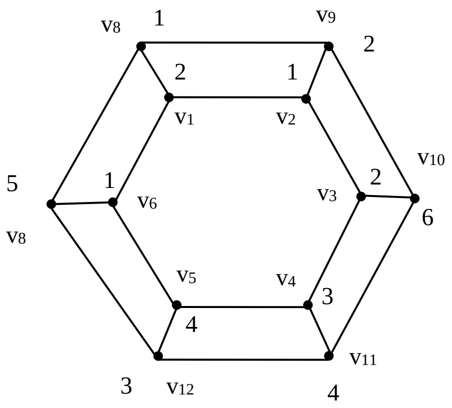

Figure 2

The vertices v_1, v_2, v_4 dominate color class 1, 2, 4 respectively. The vertices v_3, v_5 dominate color class 6, 3 respectively. The vertex v_6 dominate color class 5. The vertex v_{12} dominate color class 4 the vertices v_7, v_8 dominate color class 5 and the vertices v_9, v_{10}, v_{11} dominate color class 6. Thus $\chi_d(Y_6) = 6$.

Observation 2.3: The dominator coloring of a prism graph Y_n $when$ $n = 5$ is 6. i.e., $\chi_d(Y_5) = 6$.

Illustration 2.3: Figure 3 shows the dominator coloring of prism graph Y_5 .

The vertices v_i, $1 \leq i \leq 5$, dominate color class i. The vertices v_6, v_7 dominate color class 5 and the vertices v_{5+i}, $3 \leq i \leq 5$ dominate color class 6. Thus the minimum number of colors needed for dominator coloring of Y_5 is 6. i.e., $\chi_d(Y_5) = 6$.

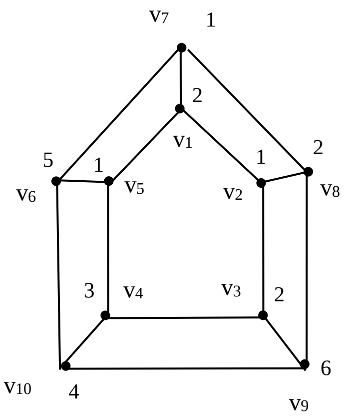

Figure 3

Theorem 2.4: For a prism graph Y_n , $n \geq 3$, $\chi_d(Y_n) > \gamma(Y_n)$

Proof:

The dominator chromatic number for a prism graph Y_n is given by $\chi_d(Y_n) = \begin{cases} n+1 & for\ n = 5, and\ n \geq 9 \\ n & for\ n \neq 5\ and\ 3 \leq n \leq 8 \end{cases}$

The domination number for a prism graph Y_n is given by [5]

$$\gamma(Y_n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv -4 \pmod 4 \\ \frac{n}{2} + 1 & \text{if } n \equiv -2 \pmod 4 \\ \frac{n+1}{2} & \text{if } n \equiv -1 \pmod 2 \end{cases}$$

Hence $\chi_d(Y_n) > \gamma(Y_n),\ \forall\, n \geq 3$

Theorem 2.5: For a prism graph Y_n, $n \geq 3$, $\chi_d(Y_n) \geq \chi(Y_n)$

Proof:

The dominator chromatic number for a prism graph Y_n is given by $\chi_d(Y_n) = \begin{cases} n+1 & for\ n = 5, and\ n \geq 9 \\ n & for\ n \neq 5\ and\ 3 \leq n \leq 8 \end{cases}$

The chromatic number for a prism graph Y_n is given by [9] $\chi(Y_n) = \begin{cases} 2 & when\ n\ is\ even \\ 3 & when\ n\ is\ odd \end{cases}$

Thus for $n > 3$, $\chi_d(Y_n) > \chi(Y_n)$.

For $n = 3$, $\chi_d(Y_n) = \chi(Y_n) = 3$. Thus the bound is sharp.

Hence $\chi_d(Y_n) \geq \chi(Y_n)$.

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