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MHD Free Convective Flow through a Porous Medium Past a Vertical Plate with Ramped Wall Temperature

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Abstract

An exact solution to the problem of the natural convective flow of an optically thin viscous incompressible electrically conducting fluid past a vertical plate in a porous medium with ramped wall temperature is obtained in presence of appreciable thermal radiation. Equations governing the flow and heat transfer are solved analytically by adopting Laplace transform technique in closed form. Expressions for the velocity field and temperature field are obtained in non-dimensional form. Effects of several parameters on the above fields are studied through graphs and tables and are physically interpreted. The application of the transverse magnetic field causes the flow to retard. It is found that velocity increases with increasing Grashof number. Moreover due to the increase in porosity parameter and magnetic field, the shear stress at the wall rises.

Keywords: MHD, Natural convection, Ramped wall temperature, Porous medium, Thermal radiation

1. Introduction

Magnetohydrodynamics is a branch of dynamics which is concerned with the study of motion of electrically conducting fluid in presence of magnetic field. In other words MHD theory is of that conductive fluids whether liquids or gaseous, which can support magnetic fields. In earlier years MHD was applied to astrophysical and geophysical problems, where it is still very important. In engineering MHD is employed to study mostly the magnetic behavior of plasmas in fusion reactors, liquid- metal cooling of nuclear reactors and electromagnetic casting. Many people have extensively studied in this field with its applications. Das et al. [17] have studied the unsteady MHD flow and heat transfer of incompressible electrically conducting viscous fluid past an infinite heated porous plate. The unsteady MHD natural convection flow and mass transfer along an accelerated porous plate in a porous medium have been studied by Sattar and Maleque [7], and Sattar et al. [8].

Natural convection or free convection is a mechanism of heat transport in which the motion of the fluid is not generated by any external source like a pump, suction device etc. but only by density differences in the fluid which occurs due to temperature gradients. Anwar [6] has worked on MHD unsteady free convective flow past a vertical porous plate. Goshdastidar [14] has done a comparative study on various fields of free convection flows and also its application. Nield and Bejan [4] have also worked in the same field. Vedhanayagam et al. [11], Martynenko et al. [12], Kolar et al. [2], Ramanaiah et al. [5] and Camargo et al. [15] have extensively studied free convection effect on flow past a vertical surface with different boundary conditions. Also Revankar [18], Sahoo et al. [13] have worked on hydromagnetic natural convection flow past a vertical surface. Thermal radiation is the transfer of energy by the emission of electromagnetic waves which carry energy away from the emitting object. The effects of thermal radiation on natural convective flow are significant in problems involving absorbing and emitting fluids. Many authors have studied experimentally and theoretically fluid flow through a porous channel because of its wide applications in many fields such as diffusion technology, transpiration cooling etc. Samad and Rahman [9] have analyzed thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in porous medium. Ibrahim et al. [10] have investigated thermal radiation effect on a porous media under optically thick approximation using Newton Scheme method from Taylor series. The flow of an incompressible viscous fluid near a porous oscillating infinite plate with suction or blowing condition was studied by Biihler and Zierep [3]. Das et al. [16] have made an analytical investigation to study the radiation effect on natural convective flow past a vertical plate in the presence of porous medium. They have used the Cogley’s et al. [1] model to describe the radiative heat transfer.

The objective of the present work is to study the free convective flow through a porous medium past a vertical plate with ramped wall temperature in presence of magnetic field. The work is an extension of the work done by Das et al. [16] to consider the effects of magnetic field which is found to be very important in controlling and regulating the fluid velocity and viscous drag at wall.

2. Basic equations

The equations governing the flow of a viscous incompressible electrically conducting and radiating fluid in the presence of magnetic field are:

Equation of continuity

$$\vec{\nabla}.\vec{q} = 0 \tag{1}$$

Momentum equation

$$\rho[\frac{\partial \vec{q}}{\partial t'} + (\vec{q}.\vec{\nabla})\vec{q}] = -\vec{\nabla} p + \rho \vec{g} + \mu \nabla^2 \vec{q} - \frac{\mu}{k^*}\vec{q} + \vec{J} \times \vec{B} \tag{2}$$

Energy equation

$$\rho C_p[\frac{\partial T}{\partial t'} + (\vec{q}.\vec{\nabla})T] = \kappa \nabla^2 T + \phi - \vec{\nabla}.\vec{q}_r \tag{3}$$

Where $\vec{q}$ is the fluid velocity, ρ the fluid density, p the fluid pressure, μ the coefficient of viscosity, $\vec{J}$ the current density, $\vec{B}$ the magnetic induction vector, $\vec{g}$ the acceleration due to gravity, κ the thermal conductivity, C_p the specific heat at constant pressure, T the fluid temperature , $\vec{q}_r$ the radiative heat flux , ϕ the viscous dissipation of energy and k^* is the permeability of porous media.

3. Mathematical analysis

An unsteady free convective flow of an optically thin viscous incompressible fluid past an infinite vertical plate moving in its own plane in presence of magnetic field is considered for investigation. We take x-axis along the wall in vertical direction and y-axis perpendicular to the wall. The flow is confined to the region y > 0 and the plate coincides with y=0. The plate and the surrounding fluid are at the same constant temperature T_w at time $t' \le 0$. At $0 < t' \le t_0$, the temperature of the wall is lowered to $T_\infty + (T_w - T_\infty)\frac{t'}{t_0}$. As the plate is infinite along x-direction, all the physical variables are functions of y and t. A uniform magnetic field of strength B_0 is applied normal to the plate. The flow is taken to be optically thin gray gas with free convection and radiation.

Our investigation is restricted to the following assumptions:

(i) Motion is one dimensional.

(ii) All the fluid properties are constant except the density in the buoyancy force term.

(iii) The radiative heat flux in x-direction is negligible as compared to that in y-direction.

(iv) Viscous dissipation of energy is considered negligible.

Equation of state on the basis of classical Boussinesq approximation is

$$\rho_\infty = \rho\left[1 + \beta^*\left(T - T_\infty\right)\right] \tag{4}$$

Where β^* is the coefficient of thermal expansion, T_∞ and ρ_∞ are the temperature and density respectively of the fluid when it is away from the plate.

Using Boussinesq approximation (4), the momentum equation (2) along x-axis reduces to

$$\frac{\partial u}{\partial t'} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta^*(T - T_\infty) - \frac{\nu}{k^*}u - \frac{\sigma B_0^{\,2} u}{\rho} \tag{5}$$

where u, ν, σ, B_0 are respectively fluid velocity, kinematic viscosity, electrical conductivity and strength of magnetic field.

The energy equation (3) on the basis of the assumptions (iii) and (iv), takes the form

$$\frac{\partial T}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{6}$$

The initial and boundary conditions are

$$y \ge 0: \quad u = 0,\ T = T_\infty \text{ for } t' \le 0 \tag{7.1}$$

$$\left.\begin{aligned} y = 0: \quad & u = U_0 \text{ for } t' > 0, \\ & T = T_\infty + (T_w - T_\infty)\frac{t'}{t_0} \text{ for } 0 < t' \le t_0 \\ & T = T_w \text{ for } t' > t_0 \end{aligned}\right\} \tag{7.2}$$

$$y \to \infty: u \to 0, T \to T_\infty \text{ for } t' > 0 \tag{7.3}$$

Using Cogley's model ([1]), the rate of radiative heat flux for a non gray gas near equilibrium in a optically thin fluid is given by

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda \tag{8}$$

where K_λ is the absorption coefficient, λ the wave length, $e_{\lambda h}$ is the Planck's function and subscript 0 implies that all physical quantities have been evaluated at the temperature T_∞.

Application of equation (8) in equation (6) results in

$$\frac{\partial T}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{4}{\rho C_p}(T - T_\infty)I \tag{9}$$

where $I = \int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda$

In order to make the mathematical model normalized, we introduce the dimensionless variables and parameters as follows:

$$\eta = \frac{y}{U_0 t_0},\ t = \frac{t'}{t_0},\ u_1 = \frac{u}{U_0},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ M = \frac{\sigma B_0^{\,2} t_0}{\rho},\ Gr = \frac{g\beta^*\nu(T - T_\infty)}{U_0^{\,3}},$$

$$Pr = \frac{\nu\rho C_p}{\kappa}\ K = \frac{k^* U_0^{\,2}}{\nu^2} = MaDa,\ t_0 = \frac{\upsilon}{U_0^{\,2}} \tag{10}$$

Where M is the Hartmann number, Gr the Grashof number, Pr the Prandtl number, K the porosity parameter and t_0 is the characteristic time.

Using (10), we arrive at the non-dimensional forms of equations (5) and (9) as

$$\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial \eta^2} + Gr\theta - \frac{1}{K}u_1 - Mu_1 \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} - Ra\theta \tag{12}$$

The corresponding initial and boundary conditions for u_1 and θ are

$$\eta \ge 0: \; u_1 = 0, \theta = 0 \text{ for } t \le 0, \tag{13.1}$$

$$\left.\begin{aligned} \eta = 0: \quad & u_1 = 1 \text{ for } t > 0, \\ & \theta = \tau \text{ for } 0 < t \le 1, \\ & \theta = 1 \text{ for } t > 1 \end{aligned}\right\} \tag{13.2}$$

$$\eta \to \infty : u_1 \to 0, \theta \to 0 \text{ for } t > 0. \tag{13.3}$$

Taking Laplace transform of (11) and (12) respectively, we get

$$\frac{d^2 \overline{u_1}}{d\eta^2} - \overline{u_1}\left(s + \frac{1+MK}{K}\right) = -Gr\overline{\theta} \tag{14}$$

$$\frac{d^2 \overline{\theta}}{d\eta^2} - Pr(s+Ra)\overline{\theta} = 0 \tag{15}$$

Where $\overline{f}(\eta, s) = \int_0^\infty f(\eta, t) e^{-st} dt$

The boundary conditions for $\overline{u_1}$ and $\overline{\theta}$ are

$$\overline{u_1} = \frac{1}{s}, \overline{\theta} = \frac{1}{s^2}\left(1 - e^{-s}\right) \text{ at } \eta = 0 \tag{16}$$

$$\overline{u_1} \to 0, \; \overline{\theta} \to 0 \text{ as } \eta \to \infty$$

Equations (15) and (14) are solved subject to the boundary conditions (16) and the solutions obtained are inverted by taking inverse Laplace transform to obtain the following results:

$$\theta(\eta, t) = \theta_1(\eta, t) - \theta_1(\eta, t-1) H(t-1) \tag{17}$$

$$u_1(\eta, t) = \frac{1}{2}\left[e^{\eta\sqrt{M_1}} erfc\left(\frac{\eta}{2\sqrt{t}} + \sqrt{M_1 t} \right) + e^{-\eta\sqrt{M_1}} erfc\left(\frac{\eta}{2\sqrt{t}} - \sqrt{M_1 t} \right) \right]$$
$$-\alpha\left[\psi_1(\eta, t) - \psi_1(\eta, t-1) H(t-1) \right] \tag{18}$$

Where

$$\theta_1(\eta,t)=\frac{1}{2}\left[F\left(Pr,Ra,\eta,t\right)+\overline{F}\left(Pr,Ra,\eta,t\right)\right] \tag{19}$$

where $F\left(Pr,Ra,\eta,t\right)=\left(t+\frac{\eta}{2}\sqrt{\frac{Pr}{Ra}}\right)e^{\eta\sqrt{PrRa}}erfc\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}}+\sqrt{tRa}\right)$

and

$$\psi_1(\eta,t)=\frac{1}{2}\left[\frac{e^{\xi t}}{\xi^2}\left\{e^{\eta\sqrt{M_1+\xi}}erfc\left(\frac{\eta}{2\sqrt{t}}+\sqrt{(M_1+\xi)t}\right)+e^{-\eta\sqrt{M_1+\xi}}erfc\left(\frac{\eta}{2\sqrt{t}}-\sqrt{(M_1+\xi)t}\right)\right.\right.$$
$$\left.-e^{\eta\sqrt{Pr(Ra+\xi)}}erfc\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}}+\sqrt{(Ra+\xi)t}\right)-e^{-\eta\sqrt{Pr(Ra+\xi)}}erfc\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}}-\sqrt{(Ra+\xi)t}\right)\right\}$$
$$-\frac{1}{\xi}\left(t+\frac{1}{\xi}+\frac{\eta}{2}\sqrt{\frac{1}{M_1}}\right)e^{\eta\sqrt{M_1}}erfc\left(\frac{\eta}{2}+\sqrt{M_1t}\right)$$
$$-\frac{1}{\xi}\left(t+\frac{1}{\xi}-\frac{\eta}{2}\sqrt{\frac{1}{M_1}}\right)e^{-\eta\sqrt{M_1}}erfc\left(\frac{\eta}{2}-\sqrt{M_1t}\right)$$
$$+\frac{1}{\xi}\left(t+\frac{1}{\xi}+\frac{\eta}{2}\sqrt{\frac{Pr}{Ra}}\right)e^{\eta\sqrt{PrRa}}erfc\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}}+\sqrt{tRa}\right)$$
$$\left.+\frac{1}{\xi}\left(t+\frac{1}{\xi}-\frac{\eta}{2}\sqrt{\frac{Pr}{Ra}}\right)e^{-\eta\sqrt{\mathrm{Pr}Ra}}erfc\left(\frac{\eta}{2}\sqrt{\frac{Pr}{t}}-\sqrt{tRa}\right)\right] \tag{20}$$

where *erfc(x)* is the complementary error function and *H(t-1)* is the unit step function and $M_1=\frac{1+MK}{K}$, $\alpha=\frac{Gr}{1-Pr}$ and $\xi=\frac{PrRa-M_1}{1-Pr}$

3.1Solution in case of unit Prandtl Number

Putting Pr=1 in Equation (15) and proceeding as before the exact solution for the fluid temperature $\theta(\eta,t)$ and fluid velocity $u_1(\eta,t)$ are obtained in following form:

$$\theta(\eta,t)=\theta_2(\eta,t)-\theta_2(\eta,t-1)H(t-1) \tag{21}$$

$$u_1(\eta,t)=\frac{1}{2}\left[e^{\eta\sqrt{M_1}}erfc\left(\frac{\eta}{2\sqrt{t}}+\sqrt{M_1t}\right)+e^{-\eta\sqrt{M_1}}erfc\left(\frac{\eta}{2\sqrt{t}}-\sqrt{M_1t}\right)\right]$$
$$-\gamma_1\left[\psi_2(\eta,t)-\psi_2(\eta,t-1)H(t-1)\right] \tag{22}$$

where

$$\theta_2(\eta,t)=\frac{1}{2}\left[\left(t+\frac{\eta}{2\sqrt{Ra}}\right)e^{\eta\sqrt{Ra}}erfc\left(\frac{\eta}{2\sqrt{t}}+\sqrt{tRa}\right)+\left(t-\frac{\eta}{2\sqrt{Ra}}\right)e^{-\eta\sqrt{Ra}}\,erfc\left(\frac{\eta}{2\sqrt{t}}-\sqrt{tRa}\right)\right] \tag{23}$$

$$\psi_2(\eta,t)=\frac{1}{2}\left[\left(t+\frac{\eta}{2\sqrt{Ra}}\right)e^{\eta\sqrt{Ra}}erfc\left(\frac{\eta}{2\sqrt{t}}+\sqrt{tRa}\right)+\left(t-\frac{\eta}{2\sqrt{Ra}}\right)e^{-\eta\sqrt{Ra}}\,erfc\left(\frac{\eta}{2\sqrt{t}}-\sqrt{tRa}\right)\right.$$
$$\left.-\left(t+\frac{\eta}{2\sqrt{M_1}}\right)e^{\eta\sqrt{M_1}}erfc\left(\frac{\eta}{2\sqrt{t}}+\sqrt{M_1t}\right)-\left(t-\frac{\eta}{2\sqrt{M_1}}\right)e^{-\eta\sqrt{M_1}}\,erfc\left(\frac{\eta}{2\sqrt{t}}-\sqrt{M_1t}\right)\right] \tag{24}$$

and $\gamma_1=\dfrac{Gr}{Ra-M_1}$

3.2 Skin friction and rate of heat transfer

The co-efficient for Skin friction τ at the wall is expressed as

$$\tau=-\left.\frac{\partial u_1}{\partial\eta}\right]_{\eta=0}=\left(\sqrt{M_1}erf\left(\sqrt{M_1t}\right)+\frac{e^{-M_1t}}{\sqrt{t\pi}}\right)+\alpha\left[\psi_3(\eta,t)-\psi_3(\eta,t-1)H(t-1)\right] \tag{25}$$

where

$$\psi_3(\eta,t)=\frac{e^{\xi t}}{\xi^2}\left\{-\sqrt{M_1+\xi}erf\left(\sqrt{(M_1+\xi)t}\right)-\frac{e^{-(M_1+\xi)t}}{\sqrt{t\pi}}+\sqrt{Pr(Ra+\xi)}erf\left(\sqrt{(Ra+\xi)t}\right)\right.$$
$$\left.+\sqrt{\frac{Pr}{t\pi}}\,e^{-(Ra+\xi)t}\right\}+\frac{1}{\xi}\left\{erf\left(\sqrt{M_1t}\right)\left(\frac{1}{2\sqrt{M_1}}+\left(t+\frac{1}{\xi}\right)\sqrt{M_1}\right)\right.$$
$$\left.-erf\left(\sqrt{tRa}\right)\left(\frac{1}{2}\sqrt{\frac{Pr}{Ra}}+\left(t+\frac{1}{\xi}\right)\sqrt{PrRa}\right)+\frac{1}{\sqrt{t\pi}}\left(t+\frac{1}{\xi}\right)\left(e^{-M_1t}-\sqrt{Pr}e^{-tRa}\right)\right\} \tag{26}$$

The rate of heat transfer at the wall in terms of Nusselt number is quantified by

$$Nu=-\left.\frac{\partial\theta}{\partial\eta}\right]_{\eta=0}=\theta_3(\eta,t)-\theta_3(\eta,t-1)H(t-1) \tag{27}$$

where

$$\theta_3(\eta,t)=-\left[\left(\frac{1}{2}\sqrt{\frac{Pr}{Ra}}+t\sqrt{PrRa}\right)erf\left(\sqrt{tRa}\right)+\sqrt{\frac{Prt}{\pi}}e^{-tRa}\right] \tag{28}$$

4. Results and Discussion

Numerical calculations for non-dimensional velocity and temperature have been carried out and plotted in Figures 1-4 for various values of, Grashof number *Gr*, , porosity parameter *K*, and Hartmann number *M*. The effect of thermal Grashof number on the fluid flow is visualized in Figure 1. From Figure 1 it is observed that Grashof number contributes in enhancing the fluid motion. Grashof number *Gr* defines the ratio of buoyancy force and viscous force. As *Gr* increases, the buoyancy force increases and viscous force becomes less effective. Subsequently the fluid flows freely. Figure 2 shows as the value of Hartmann number increases, velocity u_1 decreases. This is true because the increment of the strength of magnetic field gives rise to a resistive type of force known as Lorentz force which tends to retard the fluid motion. The outcome of Figure 2 meets this physical reality. Figure 3 shows that velocity u_1 increases for an increase in porosity parameter. Figure 4 exhibits that when *Ra* increases, the fluid temperature falls because the effect of radiation is to decrease the transport of energy to the fluid.

In order to find the effects of physical parameters *Pr*, *Gr*, *Ra*, *K*, *M* and *t* on skin friction τ and rate of heat transfer *Nu*, numerical results of it have been interpreted through tables (Table 1 and Table 2). Table 1 reveals that the value of skin friction τ decreases with increasing Prandtl number, porosity parameter, Hartmann number and radiation parameter. But with the rise in time and Grashof number the magnitude of τ increases. From Table 2 it is seen that the rate of heat transfer *Nu* increases when the values of Prandtl number, radiation parameter and time increases.

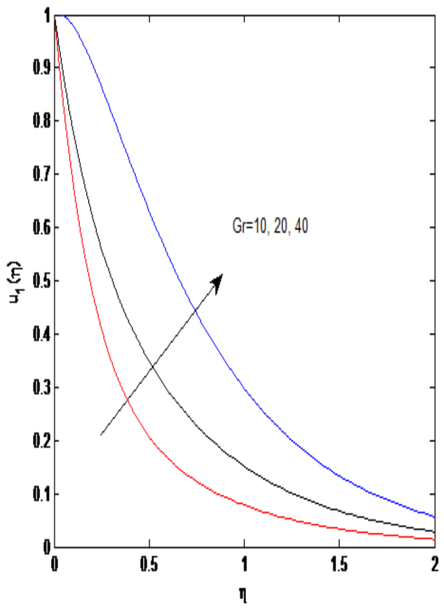


Figure1. Velocity profile for variations in Gr when Ra=2, Pr=0.71, M=5, K=0.04, t=1.0

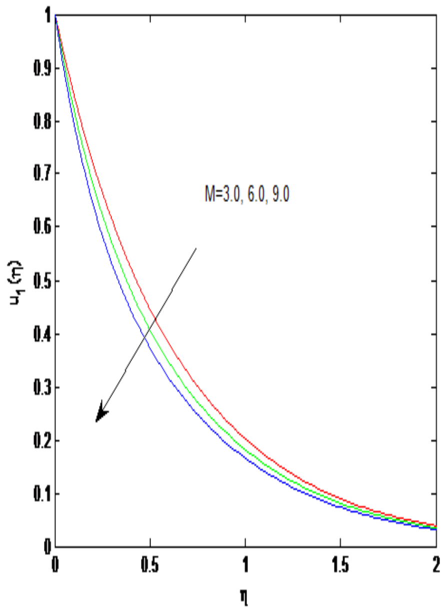
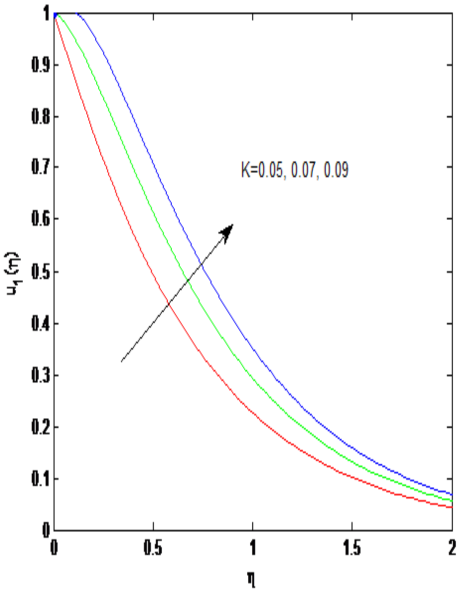


Figure 2. Velocity profile for variations in M when Ra=2, Pr=0.71, Gr=25, K=0.04, t=1.0



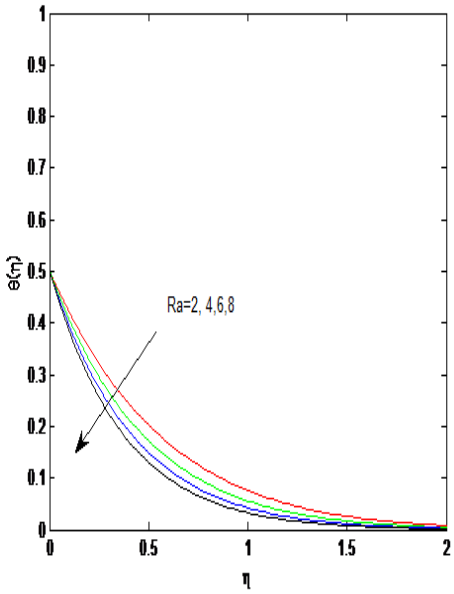


Figure 3. Velocity profile for variations in K when Ra=2, Pr=0.71,Gr=25,M=5, t=1.0

Figure 4. Temperature profile for variations in Ra when Pr=0.71, t=0.5

Table 1: Shear stress τ for different parameters.

Gr	***Pr***	***Ra***	***M***	***K***	***t***	***τ***
10	0.71	1	1	1	1	1.5184
10	0.71	1	1	0.040	1	-3.5542
10	0.71	1	1	0.045	1	-3.2066
10	0.71	1	1.5	1	1	1.3230
10	0.71	1	5	1	1	0.0425
8	0.71	1	1	1	1	0.9295
15	0.71	1	1	1	1	2.9908
10	5	1	1	1	1	0.4298
10	7	1	1	1	1	0.2297
10	0.71	2	1	1	1	1.5282
10	0.71	3	1	1	1	1.3689
10	0.71	1	1	1	1.5	2.7418
10	0.71	1	1	1	2	2.9614

Table 2: Rate of heat transfer **Nu** for different values of Pr, Ra and t

Ra	***Pr***	***t***	***Nu***
10	0.71	0.1	0.3921
2	0.71	0.1	0.3203
4	0.71	0.1	0.3392
10	**2**	0.1	0.6581
10	**5**	0.1	1.0406
10	0.71	**0.2**	0.6646
10	0.71	**0.3**	0.9322

5. Comparison

In order to validate our work, the work of Das et al. [16] is considered. Table 3 represents the numerical values of shear stress at the wall in the absence of magnetic field. It may be noted that in the special case when M=0, the values of shear stress exactly coincide with obtained by Das et al.[16] which is displayed in Table 4 (Table 2 of Das et al.[16]). The exact coincidence of results of both the work confirms the accuracy of our result.

Table 3: Shear stress τ for Pr=0.71, Ra=25, M=0 and t =1 in the present work.

Ramped temperature			
Da/Gr	**10**	**15**	**20**
0.040	3.9363	3.4044	2.8726
0.045	3.6177	3.0696	2.5214
0.050	3.3467	2.7840	2.2214
0.055	3.1124	2.5366	1.9608

Table 4: Table 2 of Das et al. [16] for shear stress $-\tau_0$ when Pr=0.71, Ra=25 and τ =1

Ramped temperature			
Da/Gr	**10**	**15**	**20**
0.040	3.9363	3.4044	2.8726
0.045	3.6177	3.0696	2.5214
0.050	3.3467	2.7840	2.2214
0.055	3.1124	2.5366	1.9608

6. Conclusions

The outcomes of the analysis are as follows:

i) The velocity of the fluid decreases in presence of magnetic field.
ii) With an increase in Grashof number, porosity parameter, fluid velocity increases.
iii) The values of skin friction decreases for increase in Prandtl number, porosity parameter, Hartmann number and radiation parameter but it rises for increased values of Grashof number and time.

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