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Direct Product of BF-Algebras

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Abstract

In this paper, we introduce the direct product of BF-algebras and we obtain some properties of this concept.

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1 Introduction

In 2007, the concept of BF-algebras was introduced by A. Walendziak [3]. A BF-*algebra* is an algebra $\mathbf{A} = (A; *, 0)$ of type $(2, 0)$, that is, a nonempty set A together with a binary operation $*$ and a constant 0, satisfying the following axioms for all $x, y \in A$:
(B1) $x * x = 0$,
(B2) $x * 0 = x$,
(BF) $0 * (x * y) = y * x$.
In [3], Walendziak also introduced the notion of commutativity of BF-algebras. A BF-algebra $\mathbf{A}$ is *commutative* if $x*(0*y) = y*(0*x)$ for all $x, y \in A$. In 2011, J.C. Endam and J.P. Vilela [2] characterized the commutativity of BF-algebras

and established the relationship of BF-algebras and groups. Walendziak also introduced the notions of subalgebras, ideals, and normality in BF-algebras, and established their properties. A subset I of A is called an *ideal* of $\mathbf{A}$ if it satisfies the following for all $x, y \in A$:
(I1) $0 \in I$,
(I2) $x * y \in I$ and $y \in I$ imply $x \in I$.
We say that an ideal I is *normal* if for any $x, y, z \in A$, $x * y \in I$ implies $(z * x) * (z * y) \in$. A nonzero ideal I of $\mathbf{A}$ is said to be *proper* if $I \neq A$. A nonempty subset N of A is called a *subalgebra* of $\mathbf{A}$ if $x*y \in N$ for any $x, y \in N$. It is known that an ideal need not be a subalgebra, and a subalgebra need not be an ideal. While a normal ideal is a subalgebra. Walendziak then used the concept of normality of BF-algebras to construct quotient BF-algebras. That is, given a normal ideal I of a BF-algebra $\mathbf{A}$, the relation $\sim_I$ is defined by $x \sim_I y$ if and only if $x * y \in I$ for any $x, y \in A$. Then $\sim_I$ is a congruence relation of $\mathbf{A}$. For $x \in A$, write x/I for the congruence class containing x, that is, $x/I = \{y \in A\colon\ x \sim_I y\}$. We denote $A/I = \{x/I\colon\ x \in A\}$ and define $*'$ by $x/I *' y/I = (x * y)/I$. Note that $x/I = y/I$ if and only if $x \sim_I y$. Then the algebra $\mathbf{A}/I = (A/I;\ *',\ 0/\text{I})$ is a BF-*algebra*. The algebra $\mathbf{A}/I$ is called the *quotient* BF-*algebra* of $\mathbf{A}$ modulo I. The concept of BF-homomorphism was also introduced by A. Walendziak. A map $\varphi : A \to B$ is called a *BF-homomorphism* if $\varphi(x * y) = \varphi(x) * \varphi(y)$ for any $x, y \in A$. The *kernel of* φ, denoted by ker φ, is defined to be the set $\{x \in A : \varphi(x) = 0_B\}$. A BF-homomorphism φ is called a *BF-monomorphism*, *BF-epimorphism*, or *BF-isomorphism* if φ is one-one, onto, or a bijection, respectively. In [3], A. Walendziak established the first isomorphism theorem for BF-algebras. In [1], J.C. Endam and J.P. Vilela established the second and third isomorphism theorems for BF-algebras. In this paper, we introduced the direct product of BF-algebras and established some of its properties.

2 Direct Product of BF-algebras

We begin with some examples of BF-algebras.

Example 2.1 [3] Let $\mathbb{R}$ be the set of real numbers and let $\mathbf{A} = (\mathbb{R};\ *,\ 0)$ be the algebra with the operation $*$ defined by

$$x * \mathrm{y} = \begin{cases} x & \text{if } y = 0, \\ y & \text{if } x = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\mathbf{A}$ is a BF-algebra.

Example 2.2 [3] Let $A = [0, \infty) = \{x \in \mathbb{R} : x \geq 0\}$. Define the binary operation $*$ on A as follows: $x * y = |x - y|$ for all $x, y \in A$. Then $(A; *, 0)$ is a BF-algebra.

Example 2.3 [3] Let $A = \{0, 1, 2, 3\}$ and $*$ be defined by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	0
2	2	3	0	2
3	3	0	2	0

Then $(A; *, 0)$ is a BF-algebra.

Let $\mathbf{A} = (A; *, 0_A)$ and $\mathbf{B} = (B; *, 0_B)$ be BF-algebras. Define the direct product of $\mathbf{A}$ and $\mathbf{B}$ to be the structure $\mathbf{A} \times \mathbf{B} = (A \times B; \circledast, (0_A, 0_B))$, where $A \times B$ is the set $\{(a, b) : a \in A \text{ and } b \in B\}$ and whose binary operation $\circledast$ is given by $(a_1, b_1) \circledast (a_2, b_2) = (a_1 * a_2, b_1 * b_2)$. Note that the binary operation $\circledast$ is componentwise. Thus, the properties (B1), (B2), and (BF) of $\mathbf{A} \times \mathbf{B}$ follow from those of $\mathbf{A}$ and $\mathbf{B}$. Hence, the following theorem easily follows.

Theorem 2.4 *The direct product of two BF-algebras is also a BF-algebra.*

Now, we extend this direct product to any finite family of BF-algebras. Let $\mathrm{I}_n = \{1, 2, \ldots, n\}$ and let $\{\mathbf{A}_i = (A_i; *, 0_i) : i \in \mathrm{I}_n\}$ be a finite family of BF-algebras. Define the direct product of BF-algebras $\mathbf{A}_1, \ldots, \mathbf{A}_n$ to be the structure $\prod_{i=1}^{n} \mathbf{A}_i = \left(\prod_{i=1}^{n} A_i; \circledast, (0_1, \ldots, 0_n) \right)$, where

$$\prod_{i=1}^{n} A_i = A_1 \times \cdots \times A_n = \{(a_1, \ldots, a_n) : a_i \in A_i, i \in \mathrm{I}_n\}$$

and whose operation $\circledast$ is given by

$$(a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) = (a_1 * b_1, \ldots, a_n * b_n).$$

Obviously, $\circledast$ is a binary operation on $\prod_{i=1}^{n} \mathbf{A}_i$.

Remark 2.5 *If* $\{\boldsymbol{A}_i = (A_i; *, 0_i)$*:* $i \in I_n\}$ *is a family of BF-algebras, then* $\prod_{i=1}^{n} \boldsymbol{A}_i$ *is a BF-algebra.*

Lemma 2.6 *Let $\{\boldsymbol{A}_i = (A_i; *, 0_i)\colon i \in I_n\}$ be a family of BF-algebras. Then each $\boldsymbol{A}_i$ is commutative if and only if $\prod_{i=1}^{n} \boldsymbol{A}_i$ is commutative.*

Proof: Let each $\mathbf{A}_i$ be commutative. If $(a_1, \ldots, a_n)$, $(b_1, \ldots, b_n) \in \prod_{i=1}^{n} A_i$, then $a_i, b_i \in A_i$ and $a_i * (0_i * b_i) = b_i * (0_i * a_i)$ for all $i \in \mathrm{I}_n$. Thus,

$$\begin{aligned}(a_1, \ldots, a_n) \circledast ((0_1, \ldots, 0_n) \circledast (b_1, \ldots, b_n)) &= (a_1, \ldots, a_n) \circledast (0_1 * b_1, \ldots, 0_n * b_n)\\ &= (a_1 * (0_1 * b_1), \ldots, a_n * (0_n * b_n))\\ &= (b_1 * (0_1 * a_1), \ldots, b_n * (0_n * a_n))\\ &= (b_1, \ldots, b_n) \circledast (0_1 * a_1, \ldots, 0_n * a_n)\\ &= (b_1, \ldots, b_n) \circledast ((0_1, \ldots, 0_n) \circledast (a_1, \ldots, a_n)).\end{aligned}$$

Therefore, $\prod_{i=1}^{n} \mathbf{A}_i$ is commutative.

Conversely, let $\prod_{i=1}^{n} \mathbf{A}_i$ be commutative. If $a_i, b_i \in A_i$ for all $i \in \mathrm{I}_n$, then $(a_1, \ldots, a_n)$, $(b_1, \ldots, b_n) \in \prod_{i=1}^{n} A_i$ and

$$\begin{aligned}&(a_1, \ldots, a_n) \circledast ((0_1, \ldots, 0_n) \circledast (b_1, \ldots, b_n))\\ &\qquad = (b_1, \ldots, b_n) \circledast ((0_1, \ldots, 0_n) \circledast (a_1, \ldots, a_n)).\end{aligned}$$

Thus,

$$\begin{aligned}(a_1 * (0_1 * b_1), \ldots, a_n * (0_n * b_n)) &= (a_1, \ldots, a_n) \circledast (0_1 * b_1, \ldots, 0_n * b_n)\\ &= (a_1, \ldots, a_n) \circledast ((0_1, \ldots, 0_n) \circledast (b_1, \ldots, b_n))\\ &= (b_1, \ldots, b_n) \circledast ((0_1, \ldots, 0_n) \circledast (a_1, \ldots, a_n))\\ &= (b_1, \ldots, b_n) \circledast (0_1 * a_1, \ldots, 0_n * a_n)\\ &= (b_1 * (0_1 * a_1), \ldots, b_n * (0_n * a_n)).\end{aligned}$$

This implies that $a_i * (0 * b_i) = b_i * (0 * a_i)$ for all $i \in \mathrm{I}_n$. Therefore, each $\mathbf{A}_i$ is commutative. □

Theorem 2.7 *Let $\{\varphi_i : A_i \to B_i\colon i \in I_n\}$ be a family of BF-homomorphisms. If φ is the map $\prod_{i=1}^{n} A_i \to \prod_{i=1}^{n} B_i$ given by $(a_1, \ldots, a_n) \mapsto (\varphi_1(a_1), \ldots, \varphi_n(a_n))$, then φ is a BF-homomorphism with $\ker \varphi = \prod_{i=1}^{n} \ker \varphi_i$, $\varphi(\prod_{i=1}^{n} A_i) = \prod_{i=1}^{n} \varphi_i(A_i)$. Furthermore, φ is a BF-monomorphism (respectively, BF-epimorphism) if and only if φ_i is.*

Proof: Let $\{\varphi_i : A_i \to B_i : i \in \mathrm{I}_n\}$ be a family of BF-homomorphisms and let φ be the map $\prod_{i=1}^{n} A_i \to \prod_{i=1}^{n} B_i$ given by $(a_1, \ldots, a_n) \mapsto (\varphi_1(a_1), \ldots, \varphi_n(a_n))$. If $(a_1, \ldots, a_n)$, $(b_1, \ldots, b_n) \in \prod_{i=1}^{n} A_i$, then

$$\begin{aligned}
\varphi((a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n)) &= \varphi((a_1 * b_1, \ldots, a_n * b_n)) \\
&= (\varphi_1(a_1 * b_1), \ldots, \varphi_n(a_n * b_n)) \\
&= (\varphi_1(a_1) * \varphi_1(b_1), \ldots, \varphi_n(a_n) * \varphi_n(b_n)) \\
&= (\varphi_1(a_1), \ldots, \varphi_n(a_n)) \circledast (\varphi_1(b_1), \ldots, \varphi_n(b_n)) \\
&= \varphi((a_1, \ldots, a_n)) \circledast \varphi((b_1, \ldots, b_n)).
\end{aligned}$$

This shows that φ is a BF-homomorphism. Moreover,

$$\begin{aligned}
(a_1, \ldots, a_n) \in \ker \varphi &\Leftrightarrow \varphi((a_1, \ldots, a_n)) = (0_1, \ldots, 0_n) \\
&\Leftrightarrow (\varphi_1(a_1), \ldots, \varphi_n(a_n)) = (0_1, \ldots, 0_n) \\
&\Leftrightarrow \varphi_i(a_i) = 0_i \text{ for each } i \in \mathrm{I}_n \\
&\Leftrightarrow a_i \in \ker \varphi_i \text{ for each } i \in \mathrm{I}_n \\
&\Leftrightarrow (a_1, \ldots, a_n) \in \prod_{i=1}^{n} \ker \varphi_i.
\end{aligned}$$

Thus, $\ker \varphi = \prod_{i=1}^{n} \ker \varphi_i$. Let $A = \prod_{i=1}^{n} A_i$. Then

$$\begin{aligned}
(b_1, \ldots, b_n) \in \varphi(A) &\Leftrightarrow \exists\ (a_1, \ldots, a_n) \in A \ni (b_1, \ldots, b_n) = \varphi((a_1, \ldots, a_n)) \\
&\Leftrightarrow \exists\ (a_1, \ldots, a_n) \in A \ni (b_1, \ldots, b_n) = (\varphi_1(a_1), \ldots, \varphi_n(a_n)) \\
&\Leftrightarrow \exists\ a_i \in A_i \ni b_i = \varphi_i(a_i) \in \varphi(A_i) \text{ for each } i \in \mathrm{I}_n \\
&\Leftrightarrow (b_1, \ldots, b_n) \in \prod_{i=1}^{n} \varphi_i(A_i).
\end{aligned}$$

Thus, $\varphi(\prod_{i=1}^{n} A_i) = \prod_{i=1}^{n} \varphi_i(A_i)$.

To prove the last statement, let φ be one-to-one. If $\varphi_i(a_i) = \varphi(b_i)$ for each $i \in \mathrm{I}_n$, then

$$\begin{aligned}
\varphi((a_1, \ldots, a_n)) &= (\varphi_1(a_1), \ldots, \varphi_n(a_n)) \\
&= (\varphi_1(b_1), \ldots, \varphi_n(b_n)) \\
&= \varphi((b_1, \ldots, b_n)).
\end{aligned}$$

Since φ is one-to-one, $(a_1, \ldots, a_n) = (b_1, \ldots, b_n)$, that is, $a_i = b_i$ for each $i \in \mathrm{I}_n$. Therefore, φ_i is one-to-one for each $i \in \mathrm{I}_n$. Conversely, let φ_i be one-to-one for each $i \in \mathrm{I}_n$. If $\varphi((a_1, \ldots, a_n)) = \varphi((b_1, \ldots, b_n))$, then

$$\begin{aligned}(\varphi_1(a_1), \ldots, \varphi_n(a_n)) &= \varphi((a_1, \ldots, a_n)) \\ &= \varphi((b_1, \ldots, b_n)) \\ &= (\varphi_1(b_1), \ldots, \varphi_n(b_n)).\end{aligned}$$

Thus, $\varphi_i(a_i) = \varphi_i(b_i)$ for each $i \in \mathrm{I}_n$. Since each φ_i is one-to-one, $a_i = b_i$ for each $i \in \mathrm{I}_n$ and so $(a_1, \ldots, a_n) = (b_1, \ldots, b_n)$. Therefore, φ is one-to-one.

Finally, we show that φ is onto if and only if each φ_i is. Let φ be onto. If $b_i \in B_i$ for each $i \in \mathrm{I}_n$, then $(b_1, \ldots, b_n) \in \prod_{i=1}^{n} B_i$. Since φ is onto, there exists $(a_1, \ldots, a_n) \in \prod_{i=1}^{n} A_i$ such that

$$(b_1, \ldots, b_n) = \varphi((a_1, \ldots, a_n)) = (\varphi_1(a_1), \ldots, \varphi_n(a_n)),$$

that is, $b_i = \varphi_i(a_i)$ for each $i \in \mathrm{I}_n$. Therefore, φ_i is onto for each $i \in \mathrm{I}_n$. Conversely, let φ_i be onto for each $i \in \mathrm{I}_n$. If $(b_1, \ldots, b_n) \in \prod_{i=1}^{n} B_i$, then $b_i \in B_i$ for each $i \in \mathrm{I}_n$. Since each φ_i is onto, there exists $a_i \in A_i$ such that $b_i = \varphi_i(a_i)$ for each $i \in \mathrm{I}_n$ so that

$$(b_1, \ldots, b_n) = (\varphi_1(a_1), \ldots, \varphi_n(a_n)) = \varphi((a_1, \ldots, a_n)).$$

Therefore, φ is onto and so the theorem is finally proved. □

Remark 2.8 *Let* $\{\boldsymbol{A}_i = (A_i; *, 0_i)\colon i \in I_n\}$, $\{\boldsymbol{B}_i = (B_i; *, 0_i)\colon i \in I_n\}$ *be any two families of BF-algebras such that* $A_i \cong B_i$ *for each* $i \in I_n$. *Then* $\prod_{i=1}^{n} A_i \cong \prod_{i=1}^{n} B_i$.

Theorem 2.9 *Let* $\{\boldsymbol{A}_i = (A_i; *, 0_i)\colon i \in I_n\}$ *be a family of BF-algebras and let* J_i *be a normal ideal of* $\boldsymbol{A}_i$ *for each* $i \in I_n$. *Then* $\prod_{i=1}^{n} J_i$ *is a normal ideal of* $\prod_{i=1}^{n} \boldsymbol{A}_i$ *and* $\prod_{i=1}^{n} A_i \,/ \prod_{i=1}^{n} J_i \cong \prod_{i=1}^{n} (A_i/J_i)$.

Proof: Let $\{\mathbf{A}_i = (A_i; *, 0_i)\colon i \in \mathrm{I}_n\}$ be a family of BF-algebras and let J_i be a normal ideal of $\mathbf{A}_i$ for each $i \in \mathrm{I}_n$. Then $(0_1, \ldots, 0_n) \in \prod_{i=1}^{n} J_i$ since $0_i \in J_i$ for each $i \in \mathrm{I}_n$ and so $\prod_{i=1}^{n} J_i$ is not empty. Let $(a_1, \ldots, a_n), (b_1, \ldots, b_n) \in \prod_{i=1}^{n} A_i$.

If $(b_1, \ldots, b_n)$, $(a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) \in \prod_{i=1}^{n} J_i$, then $b_i \in J_i$ for each $i \in \mathrm{I}_n$ and $(a_1 * b_1, \ldots, a_n * b_n) = (a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) \in \prod_{i=1}^{n} J_i$ implies that $a_i * b_i \in J_i$ for each $i \in \mathrm{I}_n$. By (I2), $a_i \in J_i$ for each $i \in \mathrm{I}_n$ and so $(a_1, \ldots, a_n)$ is an element of $\prod_{i=1}^{n} J_i$. Thus, $\prod_{i=1}^{n} J_i$ is an ideal of $\prod_{i=1}^{n} \mathbf{A}_i$.

Let $(c_1, \ldots, c_n) \in \prod_{i=1}^{n} A_i$. If $(a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) \in \prod_{i=1}^{n} J_i$, then $(a_1 * b_1, \ldots, a_n * b_n) = (a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) \in \prod_{i=1}^{n} J_i$ so that $a_i * b_i \in J_i$ for each $i \in \mathrm{I}_n$ and so $(c_i * a_i) * (c_i * b_i) \in J_i$ for each $i \in \mathrm{I}_n$. Moreover,

$$\begin{aligned}((c_1, \ldots, c_n) \circledast (a_1, \ldots, a_n)) \circledast ((c_1, \ldots, c_n) \circledast (b_1, \ldots, b_n)) \\ &= (c_1 * a_1, \ldots, c_n * a_n) \circledast (c_1 * b_1, \ldots, c_n * b_n) \\ &= ((c_1 * a_1) * (c_1 * b_1), \ldots, (c_n * a_n) * (c_n * b_n)) \in \prod_{i=1}^{n} J_i.\end{aligned}$$

Therefore, $\prod_{i=1}^{n} J_i$ is a normal ideal of $\prod_{i=1}^{n} \mathbf{A}_i$.

For simplicity, let $J = \prod_{i=1}^{n} J_i$ and $A = \prod_{i=1}^{n} A_i$. Define φ: $A/J \to \prod_{i=1}^{n}(A_i/J_i)$ given by $\varphi((a_1, \ldots, a_n)/J) = (a_1/J_1, \ldots, a_n/J_n)$ for all $(a_1, \ldots, a_n)/J \in A/J$. Let $(a_1, \ldots, a_n)/J$, $(b_1, \ldots, b_n)/J \in A/J$. If $(a_1, \ldots, a_n)/J = (b_1, \ldots, b_n)/J$, then $(a_1, \ldots, a_n) \sim_J (b_1, \ldots, b_n)$, that is,

$$(a_1 * b_1, \ldots, a_n * b_n) = (a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) \in J.$$

Thus, $a_i * b_i \in J_i$ for all $i \in \mathrm{I}_n$, that is, $a_i \sim_{J_i} b_i$ so that $a_i/J_i = b_i/J_i$. It follows that

$$\begin{aligned}\varphi((a_1, \ldots, a_n)/J) &= (a_1/J_1, \ldots, a_n/J_n) \\ &= (b_1/J_1, \ldots, b_n/J_n) \\ &= \varphi((b_1, \ldots, b_n)/J).\end{aligned}$$

This shows that φ is well-defined. If $(a_1, \ldots, a_n)/J, (b_1, \ldots, b_n)/J \in A/J$, then

$$\begin{aligned}\varphi((a_1, \ldots, a_n)/J *' (b_1, \ldots, b_n)/J) &= \varphi(((a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n))/J) \\ &= \varphi((a_1 * b_1, \ldots, a_n * b_n)/J) \\ &= ((a_1 * b_1)/J_1, \ldots, (a_n * b_n)/J_n) \\ &= (a_1/J_1 *' b_1/J_1, \ldots, a_n/J_n *' b_n/J_n) \\ &= (a_1/J_1, \ldots, a_n/J_n) \circledast (b_1/J_1, \ldots, b_n/J_n) \\ &= \varphi((a_1, \ldots, a_n)/J) \circledast \varphi((b_1, \ldots, b_n)/J).\end{aligned}$$

This shows that φ is a BF-homomorphism.
If $\varphi((a_1,\ldots,a_n)/J) = \varphi((b_1,\ldots,b_n)/J)$, then

$$\begin{aligned}(a_1/J_1,\ldots,a_n/J_n) &= \varphi((a_1,\ldots,a_n)/J)\\ &= \varphi((b_1,\ldots,b_n)/J)\\ &= (b_1/J_1,\ldots,b_n/J_n).\end{aligned}$$

Thus, $a_i/J_i = b_i/J_i$ for all $i \in \mathrm{I}_n$. Hence, $a_i \sim_{J_i} b_i$, that is, $a_i * b_i \in J_i$ for all $i \in \mathrm{I}_n$ so that $(a_1,\ldots,a_n) \circledast (b_1,\ldots,b_n) = (a_1 * b_1,\ldots,a_n * b_n) \in J$. Thus, $(a_1,\ldots,a_n) \sim_J (b_1,\ldots,b_n)$ and so $(a_1,\ldots,a_n)/J = (b_1,\ldots,b_n)/J$. This shows that φ is one-to-one.

If $(a_1/J_1,\ldots,a_n/J_n) \in \prod_{i=1}^{n}(A_i/J_i)$, then $a_i \in A_i$ for all $i \in \mathrm{I}_n$, that is, $(a_1,\ldots,a_n) \in A$. It follows that $(a_1/J_1,\ldots,a_n/J_n) = \varphi((a_1,\ldots,a_n)/J)$, where $(a_1,\ldots,a_n)/J \in A/J$. This shows that φ is onto. Therefore, φ is a BF-isomorphism, that is, $\prod_{i=1}^{n} A_i \,/\, \prod_{i=1}^{n} J_i \cong \prod_{i=1}^{n}(A_i/J_i)$. □

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