International Mathematical Forum, Vol. 6, 2011, no. 64, 3151 - 3159

Interval-Valued Fuzzy KU-Ideals in KU-Algebras

Samy M. Mostafa

Department of Pure Mathematics
Ain Shams University, Roxy, Cairo, Egypt
samymostafa@yahoo.com

Mokhtar A. Abd- Elnaby

Department of Pure Mathematics
Ain Shams University, Roxy, Cairo, Egypt
abdelnaby@hotmail.com

Osama R. Elgendy

Department of Pure Mathematics
Ain Shams University, Roxy, Cairo , Egypt
osama8769@yahoo.com

Abstract

In this paper the notion of Interval – Valued Fuzzy KU-ideals (briefly i-v fuzzy KU-ideal) in KU-algebras is introduced. Several theorems are stated and proved. The image and inverse image of i-v fuzzy KU-ideals are defined and how the homomorphic images and inverse images of i-v fuzzy KU-ideals become i-v fuzzy KU-ideals in KU-algebras is studied as well.

Mathematics subjects classification: 03B52, 06F35, 03G25, 94D05

Keywords: KU-algebras, fuzzy KU –ideals , interval-valued fuzzy KU-subalgebras, interval-valued fuzzy KU-ideals in KU-algebras

1 Introduction

In [6], C. Prabpayak and U. Leerawat studied ideals and congruences of BCC-algebras ([2], [3]) and introduce a new algebraic structure which is called KU-algebras and investigated some related properties. The concept of a fuzzy set, was introduced by Zadeh [8]. Xi [7] applied the concept of fuzzy set to BCK-algebras and gave some of its properties. Samy M. Mostafa and Mokhtar A. Abdel Naby [4] introduced fuzzy KU-ideals in KU-algebras. In [9], Zadeh made an extension of the concept of fuzzy set by an interval-valued fuzzy set (i.e., a fuzzy set with an interval-valued membership function). This interval-valued fuzzy set is referred to as i-v fuzzy set. Zadeh also constructed a method of approximate inference using his i-v fuzzy sets. In [1], Biswas defined interval-valued fuzzy subgroups and investigated some elementary properties. In this paper, using the notion of interval-valued fuzzy set by Zadeh. We introduced the concept of interval-valued fuzzy KU-ideals in KU-algebras (briefly i-v fuzzy KU-ideals in KU-algebras) and study some of their properties. We prove that every KU-ideals of KU-algebras *X* can be realized as i-v level KU-ideals of a KU-algebras *X*, then we obtain some related results which have been mentioned in the abstract.

2 Preliminaries

Definition 2.1. [6] An algebraic system $(X,*,0)$ of type (2, 0) is called a KU-algebra if it satisfying the following conditions:

(1) $(x*y)*[(y*z)*(x*z)]=0,$

(2) $0*x=x,$

(3) $x*0=0,$

(4) $x*y=0=y*x$ implies $x=y,$ for all x,y and $z\in X$.

In a KU-algebra *X*, we get $(0*0)*[(0*x)*(0*x)]=0$. It follows that $x*x=0$ for all $x\in X$. And if we put $y=0$ in (1), we obtain $z*(x*z)=0$ for all $x,z\in X$. A subset *S* of a KU-algebra *X* is called subalgebra of *X*, if $x,y\in S$, implies $x*y\in S$.

Definition 2.2. A non empty subset *I* of a KU-algebra *X* is said to be a KU-ideal of *X* if it satisfies:

(K_1) $0\in I$,

(K_2) $x*(y*z)\in I$ and $y\in I$ imply $x*z\in I$ for all x,y and $z\in X$.

Lemma 2.3. [4] In KU-algebra $(X,*,0)$, the following is hold

(i) $x\le y$ implies $y*z\le x*z$,

(ii) $z*(y*x)=y*(z*x)$,

(iii) $y*[(y*x)*x]=0$. for all x,y and $z\in X$.

Example 2.4. Let *X* = {0,1,2,3,4} be a set with a binary operation $*$ defined by the following table:

$*$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	0
2	0	1	0	3	0
3	0	0	0	0	0
4	0	1	0	3	0

Then $(X,*,0)$ is a KU-algebra.

3 Fuzzy KU-ideal

Definition 3.1. [4] Let X be a KU-algebra. A fuzzy set μ in X is called a fuzzy KU-ideal of X if it satisfies:

(FK_1) $\mu(0) \geq \mu(x)$,

(FK_2) $\mu(x * z) \geq \min\{\mu(x*(y*z)), \mu(y)\}$, for all $x, y, z \in X$.

Lemma 3.2. [4] Let μ be a fuzzy KU-ideal of KU-algebra X. if the inequality $x * y \leq z$ hold in X, then $\mu(y) \geq \min\{\mu(x), \mu(z)\}$.

Lemma 3.3. [4] If μ be a fuzzy KU-ideal of KU-algebra X and if $x \leq y$, then $\mu(x) \geq \mu(y)$.

Definition 3.4. Let μ be a fuzzy set on a KU-algebra X, then μ is called a fuzzy KU-subalgebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Definition 3.5. [5] Let f be a mapping from the set X to a set Y. If μ is a fuzzy subset of X, then the fuzzy subset B of Y defined by

$$f(\mu)(y) = B(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

is called the image of μ under f.

Similarly, if B is a fuzzy subset of Y, then the fuzzy subset defined by $\mu(x) = B(f(x))$ for all $x \in X$, is said to be the preimage of B under f.

Definition 3.6. Let $(X,*,0)$ and $(Y,*',0')$ be KU-algebras. A mapping $f : X \to Y$ is said to be a homomorphism if

$f(x * y) = f(x) *' f(y)$ for all $x, y \in X$.

Theorem 3.7. An into homomorphic preimage of a fuzzy KU-ideal is also fuzzy KU-ideal.

Proof. Let $f : X \to X'$ be an into homomorphism of KU-algebras, B a fuzzy KU-ideal of X' and μ the preimage of B under f. then $B(f(x)) = \mu(x)$, for all $x \in X$, (FK_1) hold, since $\mu(0) = B(f(0)) \geq B(f(x)) = \mu(x)$,

Let $x, y, z \in X$, then $\mu(x * z) = B(f(x * z)) = B(f(x) *' f(z)) \geq$

$$\min\{B(f(x) * (f(y) * f(z))), B(f(y))\} = \min\{B(f(x * (y * z))), B(f(y))\}$$
$$= \min\{\mu(x * (y * z)), \mu(y)\}.$$

Hence $\mu(x) = B(f(x)) = (B \circ f)(x)$ is a fuzzy KU-ideal of X.

Theorem 3.8. Let $f : X \to Y$ be a homomorphism between KU-algebras X and Y. For every fuzzy KU-ideal μ in X, $f(\mu)$ is a fuzzy KU-ideal of Y.

Proof. By definition $B(y') = f(\mu)(y') := \sup\limits_{x \in f^{-1}(y')} \mu(x)$ for all $y' \in Y$ and $\sup \phi := 0$

We have to prove that

$B(x' * z') \geq \min\{B(x' * (y' * z')), B(y')\}$, for all $x', y', z' \in Y$.

(i) Let $f : X \to Y$ be an onto homomorphism of KU-algebras. Let μ be a fuzzy KU-ideal of X with sup property and B the image of μ under f. Since μ is a fuzzy KU-ideal of X, we have $\mu(0) \geq \mu(x)$, for all $x \in X$. Note that $0 \in f^{-1}(0')$, where 0 and $0'$ are the zeroes elements of X and Y respectively. Thus, $B(0') = \sup_{t \in f^{-1}(0')} \mu(t) = \mu(0) \geq \mu(x)$, for all $x \in X$, which implies that $B(0') \geq \sup_{t \in f^{-1}(x')} \mu(t) = B(x')$, for any $x' \in Y$. For any $x', y', z' \in Y$, let $x_0 \in f^{-1}(x'), y_0 \in f^{-1}(y'), z_0 \in f^{-1}(z')$ be Such that

$\mu(x_0) = \sup_{t \in f^{-1}(x')} \mu(t)$, $\mu(y_0) = \sup_{t \in f^{-1}(y')} \mu(t)$ and $\mu(z_0) = \sup_{t \in f^{-1}(z')} \mu(t)$

and

$\mu(x_0 * (y_0 * z_0)) = B\{f(x_0 * (y_0 * z_0))\} = B(x' * (y' * z'))$

$= \sup_{x_0 * (y_0 * z_0) \in f^{-1}(x' * (y' * z'))} \mu(x_0 * (y_0 * z_0))\}$ $= \sup_{t \in f^{-1}((x' * y') * z')} \mu(t)$. Then

$B(x' * z') = \sup_{t \in f^{-1}(x' * z')} \mu(t) = \mu(x_0 * z_0) \geq \min\{\mu(x_0 * (y_0 * z_0)), \mu(y_0)\} =$

$\min\left\{ \sup_{t \in f^{-1}(x' * (y' * z'))} \mu(t), \sup_{t \in f^{-1}(y')} \mu(t) \right\} = \min\{B(x' * (y' * z')), B(y')\}$.

Hence B is a fuzzy KU-ideal of Y.

(ii) if f is not onto. For every $x' \in Y$ we define $X_{x'} := f^{-1}(x')$. Since f is a homomorphism

We have $(X_{x'} * X_{y'}) * X_{z'} \subset X_{x' * (y' * z')}$ for all $x', y', z' \in Y$ ……………….(v).

Let $x', y', z' \in Y$ be an arbitrary given. If $x' * (y' * z') \notin \mathrm{Im}(f) = f(X)$, then by definition

$B(x' * (y' * z')) = 0$. But if $x' * (y' * z') \notin f(X)$ .i.e. $X_{x' * (y' * z')} = \phi$, then by (v) at least one of x', y' and $z' \notin f(X)$, and hence $B(x' * z') \geq 0 = \min\{B(x' * (y' * z')), B(y')\}$.

4 Interval-valued fuzzy KU-ideal (subalgebra) of KU-algebra

An i-v fuzzy set A on the set X $(\neq \phi)$ is given by $A = \{(x, [\mu_A^L(x), \mu_A^U(x)]), x \in X\}$. (briefly, it denoted by $A = [\mu_A^L, \mu_A^U]$ where μ_A^L and μ_A^U are any two fuzzy sets in X such that $\mu_A^L \leq \mu_A^U$. Let $\tilde{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$, and let $D[0,1]$ denoted the family of all closed sub-interval of $[0,1]$. It is clear if $\mu_A^L(x) = \mu_A^U(x) = c$, where $0 \leq c \leq 1$, then $\tilde{\mu}_A(x) = [c, c]$ is in $D[0,1]$. Thus $\tilde{\mu}_A(x) \in [0,1]$, for all $x \in X$. Then the i-v fuzzy set A is given by $A = \{(x, \tilde{\mu}_A(x)), x \in X\}$, where $\tilde{\mu}_A : X \to D[0,1]$.

Now we define the refined minimum (briefly r min) and order "$\leq$" on elements $D_1 = [a_1, b_1]$ and $D_1 = [a_2, b_2]$ of $D[0,1]$ as follows:
$r\min(D_1, D_2) = [\{\min\{a_1, a_2\}, \min\{b_1, b_2\}], D_1 \leq D_2 \Leftrightarrow a_1 \leq a_2$ and $b_1 \leq b_2$.
Similarly we can define $\geq$ and $=$.

In what follows, let X denote a KU-algebra unless otherwise specified, we begin with the following definition.

Definition 4.1. An i-v fuzzy set A in X is called an i-v fuzzy KU-subalgebra of X if $\tilde{\mu}_A(x * y) \geq r\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$, for all $x, y \in X$.

Example 4.2 Let $X = \{0,1,2,3,4\}$ as in example 2.4. Define $\tilde{\mu}_A(x)$ as follows:
$\tilde{\mu}_A(x) = \begin{cases} [0.3,0.9] & \text{if } x = \{0,1,2\} \\ [0.1,0.6] & \text{otherwise} \end{cases}$. It is easy to check that A is an i-v fuzzy KU-subalgebra.

Lemma 4.3. If A is an i-v fuzzy KU-subalgebra of X, then $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, for all $x \in X$.

Proof. For every $x \in X$, we have $\tilde{\mu}_A(0) = \tilde{\mu}_A(x * x) \geq r\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(x)\} =$
$= r\min\{[\mu_A^L(x), \mu_A^U(x)], [\mu_A^L(x), \mu_A^U(x)]\} = r\min\{[\mu_A^L(x), \mu_A^U(x)]\} = \tilde{\mu}_A(x).$

Lemma 4.4. Let A be an i-v fuzzy KU-subalgebra of X. if there exist a sequence $\{X_n\}$ in X such that $\lim_{n\to\infty} \tilde{\mu}_A(x_n) = [1,1]$, then $\tilde{\mu}_A(0) = [0,1]$.

Proof. By lemma 4.3, we have $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, for all $x \in X$. Thus
$\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x_n)$ for every positive integer n, consider
$[1,1] \geq \tilde{\mu}_A(0) \geq \lim_{n\to\infty} \tilde{\mu}_A(x_n) = [1,1]$. Hence $\tilde{\mu}_A(0) = [0,1]$.

Definition 4.5. An interval-valued fuzzy set $A = \{(x, \tilde{\mu}_A(x)), x \in X\}$ in KU-algebra X is called an interval-valued fuzzy KU-ideal (i-v fuzzy KU-ideal, in short) if it satisfies the following conditions:

(1) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$,

(2) $\tilde{\mu}_A(x * z) \geq r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}$, for all $x, y, z \in X$.

Example 4.6. Let $X = \{0,1,2,3,4\}$ as in example 2.4. Define $\tilde{\mu}_A(x)$ as follows:
$\tilde{\mu}_A(x) = \begin{cases} [0.3,0.9] & \text{if } x = \{0,1,2,3\} \\ [0.1,0.6] & \text{otherwise} \end{cases}$.

It is easy to check that A is an i-v fuzzy KU-ideal of X.

Theorem 4.7. An i-v fuzzy set $A = [\mu_A^L, \mu_A^U]$ in X is an i-v fuzzy KU-ideal of X if and only if μ_A^L and μ_A^U are fuzzy KU-ideal of X.

Proof. Let μ_A^L and μ_A^U are fuzzy KU-ideal of X and $x, y, z \in X$. Consider

$$\begin{aligned}
\tilde{\mu}_A(x * z) &= [\mu_A^L(x * z), \mu_A^U(x * z)] \\
&\geq [\min\{\mu_A^L(x * (y * z)), \mu_A^L(y)\}, \min\{\mu_A^U(x * (y * z)), \mu_A^U(y)\}] \\
&= r\min\{[\mu_A^L(x * (y * z)), \mu_A^U(x * (y * z))], [\mu_A^L(y), \mu_A^U(y)]\} \\
&= r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}.
\end{aligned}$$

Conversely, suppose A is an i-v fuzzy KU-ideal of X. For any $x, y, z \in X$ we have $[\mu_A^L(x*z), \mu_A^U(x*z)] = \tilde{\mu}_A(x*z) \geq r\min\{\tilde{\mu}_A(x*(y*z)), \tilde{\mu}_A(y)\} =$
$= r\min\{[\mu_A^L(x*(y*z)), \mu_A^U(x*(y*z))], [\mu_A^L(y), \mu_A^U(y)]\}$
$= [\min\{\mu_A^L(x*(y*z)), \mu_A^L(y)\}, \min\{\mu_A^U(x*(y*z)), \mu_A^U(y)\}]$ There fore
$\mu_A^L(x*z) \geq \min\{\mu_A^L(x*(y*z)), \mu_A^L(y)\}$ and $\mu_A^U(x*z) \geq \min\{\mu_A^U(x*(y*z)), \mu_A^U(y)\}$.
Hence we get that μ_A^L and μ_A^U are fuzzy KU-ideal of X.

Theorem 4.8. Let A_1 and A_2 be i-v fuzzy KU-ideals of X. Then $A_1 \cap A_2$ is an i-v fuzzy KU-ideal of X.

Proof. $\tilde{\mu}_{A_1\cap A_2}(0) = [\mu^L_{A_1\cap A_2}(0), \mu^U_{A_1\cap A_2}(0)] \geq [\mu^L_{A_1\cap A_2}(x), \mu^U_{A_1\cap A_2}(x)] = \tilde{\mu}_{A_1\cap A_2}(x)$.
Let $x, y, z \in A_1 \cap A_2$ be, such that $x*(y*z) \in \{A_1, A_2\}$ and $y \in \{A_1, A_2\}$. Since A_1 and A_2 are i-v fuzzy KU-ideal of X, then by the theorem 4.7. We get

$$\begin{aligned}\tilde{\mu}_{A_1\cap A_2}(x*z) &= [\mu^L_{A_1\cap A_2}(x*z), \mu^U_{A_1\cap A_2}(x*z)] \\ &= [\min\{\mu^L_{A_1}(x*z), \mu^L_{A_2}(y)\}, \min\{\mu^U_{A_1}(x*z), \mu^U_{A_2}(y)\}] \\ &\geq [\min\{\mu^L_{A_1\cap A_2}(x*(y*z)), \mu^L_{A_1\cap A_2}(y)\}, \min\{\mu^U_{A_1\cap A_2}(x*(y*z)), \mu^U_{A_1\cap A_2}(y)\}] \\ &= r\min\{\tilde{\mu}_{A_1\cap A_2}(x*(y*z)), \tilde{\mu}_{A_1\cap A_2}(y)\}.\end{aligned}$$

Corollary 4.9. Let $\{A_i \mid i \in \Lambda\}$ be a family of i-v fuzzy KU-ideal of X. Then $\bigcap_{i\in\Lambda} A_i$ is also an i-v fuzzy KU-ideal of X.

Theorem 4.10. Let A be an i-v fuzzy set in X. Then A is an i-v fuzzy KU-ideal of X if and only if the non empty set $\tilde{U}(A;[\delta_1,\delta_2]) := \{x \in X \mid \tilde{\mu}_A(x) \geq [\delta_1,\delta_2]\}$ is a KU-ideal of X for every $[\delta_1,\delta_2] \in D[0,1]$. We call $\tilde{U}(A;[\delta_1,\delta_2])$ the i-v level KU-ideal of A.

Proof. Assume that A is an i-v fuzzy KU-ideal of X and let $[\delta_1,\delta_2] \in D[0,1]$ be such that $x*(y*z)$, $y \in \tilde{U}(A;[\delta_1,\delta_2])$, then
$\tilde{\mu}_A(x*z) \geq r\min\{\tilde{\mu}_A(x*(y*z)), \tilde{\mu}_A(y)\} \geq$
$r\min\{[\delta_1,\delta_2],[\delta_1,\delta_2]\} = [\delta_1,\delta_2]$ and so $x*z \in \tilde{U}(A;[\delta_1,\delta_2])$. Thus
$\tilde{U}(A;[\delta_1,\delta_2])$ is
KU-ideal of X.

Conversely, assume that $\tilde{U}(A;[\delta_1,\delta_2])$ $(\neq \phi)$ is a KU-ideal of X for every
$[\delta_1,\delta_2] \in D[0,1]$ .Suppose that there exist $x_0, y_0, z_0 \in X$ such that
$\tilde{\mu}_A(x_0*z_0) < r\min\{\tilde{\mu}_A(x_0*(y_0*z_0)), \tilde{\mu}_A(y_0)\}$. Let $\tilde{\mu}_A(x_0*(y_0*z_0)) = [\gamma_1,\gamma_2]$,
$\tilde{\mu}_A(y_0) = [\gamma_3,\gamma_4]$ and $\tilde{\mu}_A(x_0*z_0) = [\delta_1,\delta_2]$. Then
$[\delta_1,\delta_2] < r\min\{[\gamma_1,\gamma_2],[\gamma_3,\gamma_4]\}$. Taking
$[\lambda_1,\lambda_2] = \frac{1}{2}\{\tilde{\mu}_A(x_0*z_0) + r\min\{\tilde{\mu}_A(x_0*(y_0*z_0)), \tilde{\mu}_A(y_0)\}\} =$
$\frac{1}{2}([\delta_1,\delta_2] + \{\min\{\gamma_1,\gamma_2\}, \min\{\gamma_3,\gamma_4\}\}) = \frac{1}{2}[(\delta_1 + \min\{\gamma_1,\gamma_3\}), (\delta_2 + \min\{\gamma_2,\gamma_4\})]$
It follows that $\min\{\gamma_1,\gamma_3\} > \lambda_1 = \frac{1}{2}(\delta_1 + \min\{\gamma_1,\gamma_3\}) > \delta_1$ and

$\min\{\gamma_2,\gamma_4\} > \lambda_2 = \frac{1}{2}(\delta_2 + \min\{\gamma_2,\gamma_4\}) > \delta_2$, so that
$[\min\{\gamma_1,\gamma_3\}), \min\{\gamma_2,\gamma_4\}] > [\lambda_1,\lambda_2] > [\delta_1,\delta_2] = \tilde{\mu}_A(x_0 * z_0)$. There fore
$x_0 * z_0 \notin \tilde{U}(A;[\lambda_1,\lambda_2])$. On the other hand
$\tilde{\mu}_A(x_0 * (y_0 * z_0)) = [\gamma_1,\gamma_2] \geq [\min\{\gamma_1,\gamma_3\}, \min\{\gamma_2,\gamma_4\}] > [\lambda_1,\lambda_2]$.
$\tilde{\mu}_A(y_0) = [\gamma_3,\gamma_4] \geq [\min\{\gamma_1,\gamma_3\}, \min\{\gamma_2,\gamma_4\}] > [\lambda_1,\lambda_2]$, and so
$x_0 * (y_0 * z_0)$, $y_0 \in \tilde{U}(A;[\lambda_1,\lambda_2])$. It contradicts that $\tilde{U}(A;[\lambda_1,\lambda_2])$ is a KU-ideal of *X*.
Hence $\tilde{\mu}_A(x * z) \geq r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}$ for all $x, y, z \in X$. This complete the proof.

Theorem 4.11. Every KU-ideal of *X* can be realized as an i-v level KU-ideal of an i-v fuzzy KU-ideal of *X*.

Proof. Let *Y* be a KU-ideal of *X* and let *A* be an i-v fuzzy set on *X* defined by
$\tilde{\mu}_A(x) = \begin{cases} [\alpha_1,\alpha_2] & \text{if } x \in Y \\ [0,0] & \text{other wise} \end{cases}$. Where $\alpha_1, \alpha_2 \in (0,1]$ with $\alpha_1 < \alpha_2$.

It is clear that $\tilde{U}(A;[\alpha_1,\alpha_2]) = Y$. We show that *A* is an i-v fuzzy KU-ideal of *X*.
Let $x, y, z \in X$. If $(x * (y * z))$, $y \in Y$, then $x * z \in Y$, and so
$\tilde{\mu}_A(x * z) \geq [\alpha_1,\alpha_2] = r\min\{[\alpha_1,\alpha_2],[\alpha_1,\alpha_2]\} = r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}$.
If $(x * (y * z))$, $y \notin Y$, then $\tilde{\mu}_A(x * (y * z)) = [0,0] = \tilde{\mu}_A(y)\}$ and thus
$\tilde{\mu}_A(x * z) \geq [0,0] = r\min\{[0,0],[0,0]\} = r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}$.
If $x * (y * z) \in Y$ and $y \notin Y$, then $\tilde{\mu}_A(x * z) \geq [0,0] = r\min\{[0,0],[0,0]\} =$
$= r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}$.
Similarly for the case $x * (y * z) \notin Y$ and $y \in Y$ we get
$\tilde{\mu}_A(x * z) \geq r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\}$.
Therefore *A* is an i-v fuzzy KU-ideal of *X*, the proof is complete.

Theorem 4.12. Let *Y* be a subset of *X* and let *A* be an i-v fuzzy set on *X* defined by
$\tilde{\mu}_A(x) = \begin{cases} [\alpha_1,\alpha_2] & \text{if } x \in Y \\ [0,0] & \text{other wise} \end{cases}$.

If *A* is an i-v fuzzy KU-ideal *X*, then *Y* is KU-ideal of *X*.

Proof. Assume that *A* is an i-v fuzzy KU-ideal of *X*, and let $(x * y) * z$, $y \in Y$, then
$\tilde{\mu}_A(x * (y * z)) = [\alpha_1,\alpha_2] = \tilde{\mu}_A(y)$, and so $\tilde{\mu}_A(x * z) \geq r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\} =$
$= r\min\{[\alpha_1,\alpha_2],[\alpha_1,\alpha_2]\} = [\alpha_1,\alpha_2]$, this implies that $x * z \in \text{Y}$.
Hence *Y* is KU-ideal of *X*.

Theorem 4.13. If *A* is an i-v fuzzy KU-ideal of *X*, then the set $X_{\tilde{\mu}_A} := \{x \in X \mid \tilde{\mu}_A(x) = \tilde{\mu}_A(0)\}$ is a KU-ideal of *X*.

Proof. Let $x * (y * z)$, $y \in X_{\tilde{\mu}_A}$. Then $\tilde{\mu}_A(x * (y * z)) = \tilde{\mu}_A(0) = \tilde{\mu}_A(y)$, and so
$\tilde{\mu}_A(x * z) \geq r\min\{\tilde{\mu}_A(x * (y * z)), \tilde{\mu}_A(y)\} = r\min\{\tilde{\mu}_A(0), \tilde{\mu}_A(0)\} = \tilde{\mu}_A(0)$.

Combining this with condition (1) of definition 4.5. we get $\tilde{\mu}_A(x*z)=\tilde{\mu}_A(0)$, that is $x*z\in X_{\tilde{\mu}_A}$. Hence $X_{\tilde{\mu}_A}$ is a KU-ideal of *X*.

Definition 4.14. [1] Let $f: X\to Y$ be a mapping from set *X* into a set *Y*. let *B* be i-v fuzzy set in *Y*. Then the inverse image of *B*, denoted by $f^{-1}(B)$, is i-v fuzzy set in *X* with the membership function given by $\mu_{f^{-1}(B)}(x)=\tilde{\mu}_B(f(x))$, for all $x\in X$.

Lemma 4.15. [1] Let f be a mapping from set *X* into a set *Y* let $m=[m^L, m^u]$, and $n=[n^L, n^u]$ be i-v fuzzy set in *X* and *Y* respectively. Then

(1) $f^{-1}(n)=[f^{-1}(n^L), f^{-1}(n^u)]$,

(2) $f(m)=[f(m^L), f(m^u)]$.

Theorem 4.16. Let f be homomorphism from a KU-algebra *X* into a KU-algebra *Y*. If *B* is an i-v fuzzy KU-ideal of *Y*, then the inverse image $f^{-1}(B)$ of *B* is an i-v fuzzy KU-ideal of *X*.

Proof. Since $B=[\mu_B^L, \mu_B^u]$ is an i-v fuzzy KU-ideal of *Y*, it follows that from theorem 4.7, that μ_B^L and μ_B^u are fuzzy KU-ideal of *Y*. Using theorem 3.7, we know that $f^{-1}(\mu_B^L)$ and

$f^{-1}(\mu_B^u)$ are fuzzy KU-ideal of *X*. hence by lemma 4.15, we conclude that

$f^{-1}(B)=[f^{-1}(\mu_B^l), f^{-1}(\mu_B^u)]$ is an i-v fuzzy KU-ideal of *X*.

Definition 4.17. [1] Let f be a mapping from a set *X* into a set *Y*. let *A* be a an i-v fuzzy set in *X*. then the image of *A*, denoted by *f* [*A*], is the i-v fuzzy set in *Y* with membership function denoted by

$$\tilde{\mu}_{f[A]}(y):=\begin{cases}\sup\limits_{z\in f^{-1}(y)} \tilde{\mu}_A(z), \text{ if } f^{-1}(y)\neq\phi, & y\in Y\\ [0,0] & \text{otherwise}\end{cases},$$

where $f^{-1}(y):=\{x\in X \mid f(x)=y\}$.

Theorem 4.18. Let f be a homomorphism from a KU-algebra *X* into a KU-algebra *Y*. If *A* is an i-v fuzzy KU-ideal of *X*, then *f* [*A*] of *A* is an i-v fuzzy KU-ideal of *Y*.

Proof. Assume that *A* is an i-v fuzzy KU-ideal of *X*. Note that $A=[\mu_A^L, \mu_A^u]$ is an i-v fuzzy KU-ideal of *X*. It follows from theorem 3.7, that the images $f(\mu_A^L)$ and $f(\mu_A^u)$ are fuzzy KU-ideal of *Y*. Combining theorem 4.7, and lemma 4.15, we conclude that $f(A)=[f(\mu_A^L), f(\mu_A^u)]$ is an i-v fuzzy KU-ideal of *Y*.

References

[1] R. Biswas, Rosenfeld’s fuzz subgroups with interval valued membership function, Fuzzy Sets and Systems 63 (1994), No. 1 87-90.

[2] W. A. Dudek and X. Zhang, On ideal and congruences in BCC-algebras, Czechoslovak Math. Journal,48(1998), No. 123, 21-29.

[3] W. A. Dudek, On proper BCC-algebras, Bull. Ins. Math. Academic Science, 20(1992),137-150.

[4] Samy M. Mostafa, Mokhtar A. Abdel Naby and Moustafa M. M. Yousef, Fuzzy KU-ideals in KU-algebras. Submitted.

[5] Samy M. Mostafa, Fuzzy implicative ideal in BCK-algebras, Fuzzy Sets and Systems 87 (1997),361-368.

[6] C. Prabpayak and U. Leerawat, On ideals and congruences in KU-algebras, Scientia Magna Journal, 5(2009), No. 1, 54-57.

[7] O. G. Xi, Fuzzy BCK-algebra, Math. Japon. 36 (1991) 935-942.

[8] L. A. Zadeh, Fuzzy sets, Inform. And Control 8 (1965) 338-353.

[9] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning. I, Information Sci. And Control 8(1975) Z199-249.

Received: May, 2011